ECONOMIC CONSEQUENCES OF EQUITY COMPENSATION DISCLOSURE

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Abstract

The primary role of equity compensation is to provide incentives to an effort-averse agent. Here, we show that the chosen level of equity incentives, when publicly disclosed, will also convey information about future earnings, causing two-way linkages between incentive compensation and financial reporting. If either (a) market prices respond more (less) to information, (b) the manager is more (less) risk-averse, (c) earnings are more (less) noisy, then the firm’s owners choose more pronounced (muted) incentives, in turn leading to greater (lower) future earnings. The model explains observed spurious correlations between firm performance and executive compensation, and provides several new predictions linking managerial, earnings and market determinants to optimal equity holdings.

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Over the last decades and following several amendments to SEC Regulation S-K, information available to outside investors about managers’ equity ownership has become more comprehensive. Managers are required by law to disclose any trade or option exercise that would affect their equity ownership and firms must disclose new option or stock grants. That such public disclosures could convey information to outside investors is clear: owners informed on the quality of projects may choose to offer very different levels of incentives to their manager and, thus, investors should be able to make some inferences from contract disclosures. As compared to the standard environment where managerial contracts sole purpose is to elicit effort, this “capital-market” role for incentive contracts may distort the choice of incentives as well as the equilibrium level of effort and the pay level.

This paper formally examines the economic consequences of public disclosures of managerial equity incentives. In our model, an initial owner has private information on the quality of projects, which he cannot credibly disclose. The owner sells the firm and (contrary to common signaling models) cannot signal quality by retaining ownership.\(^1\) However, the owner employs a manager who may provide effort and observes the quality of the project. The managerial contract has two functions: to elicit effort from the manager at the lowest cost, and to indicate to new investors the quality of existing projects. Examining these two functions, we describe the distortions to the optimal compensation contract that this signaling process entails and provide several links between (i) the quality of information available to outside investors, (ii) the stock response to a management contract, (iii) the firm’s future operating performance, and (iv) characteristics of the managerial contract such as total pay and the level of equity incentives.

Understanding how much information about future prospects is conveyed through managerial stock ownership is of interest as part of the broader literature that examines the interactions between contract design and firm performance, on the one hand, and the cost and benefits of disclosure, on the other hand. An extensive empirical literature has examined the association between changes in compensation plans, and stock market reactions and/or future operating performance (e.g., Watts and Smith (1992), Gaver, Gaver and Battistel (1992), Dechow, Hutton and Sloan (1996), Core and Larcker (2002), Hanlon, Rajgopal and Shevlin (2003)). Yet, we are not aware

\(^1\)In this respect, the model differs from signaling models in which the original owner retains, ex-post, “skin” in the game (e.g., Leland and Pyle (1977), Hughes (1986), Kanodia and Lee (1998)).
of many theories that can shed light on the endogenous association between contracts and performance, and could provide testable predictions. In this respect, the theory allows us to pin down the underlying characteristics of the informational environment - and more specifically, the nature of informational asymmetries between insiders and outsiders - that are likely to affect the association between compensation and performance observable to outsiders.

We characterize two possible distortions that may occur in equilibrium. First, if outside investors have precise information (i.e., informational asymmetries between insiders and outsiders are low), signalling motives will imply that the owner offers more pronounced incentives and, as a consequence, higher effort and higher pay. The intuition for this result is as follows. Earnings in the more profitable firms are more sensitive to effort, thus increasing incentive pay raises earnings more for more productive firms than for less productive firms. The difference in chosen effort, in turn, can allow market forces to distinguish between firms with different productivities.

Second, if outside investors have imprecise information (i.e., informational asymmetries between insiders and outsiders are high), the initial owner signals the firm’s prospects by offering a managerial contract with muted incentives, lower effort and lower total pay. By contrast to the previous case, market prices are not sufficiently responsive to information to prevent the low-productivity from replicating the contract offered by the high-productivity firms. On the other hand, the owner of the high-productivity firm can choose muted incentives and, in doing so, reduce the potential value attainable by the low-productivity firm when pretending to be high-productivity.

**Related Literature**

Several papers in the agency literature have examined how financial markets may interact with managerial moral hazard problems. In a model with moral hazard and financial markets, Holmström and Tirole (1993) explain that organizations encourage acquisition of information by strategic traders on the market, so that this information may be then used as part of the incentive scheme. In their model, information flows are essentially one-directional (from markets to contracts), while we also capture here how information flows back from contracts to the market. Baiman and Verrecchia (1995, 1996) develop models in which insider trading may interact with managerial incentives. However, unlike in our study, their focus is not on private information held by the principal
when the contract is designed. In this respect, our model contributes to the moral hazard literature by considering how the choice of the contract may convey information about future value.

Another set of papers in the financial reporting literature have explored the signalling implications of particular disclosures required by the SEC. Sapra (2002) focuses on the effects of disclosures of hedging positions, and shows that such disclosures would lead to insufficient hedging of future risks in order to signal higher expected value. Arya and Mittendorf (2005) consider a model in which prospective agents have private information about their productivity, and show that a menu of option contracts helps the principal screen out low-productivity agents. Hayes and Schaefer (2008) examine a model in which managerial quality is unobservable, but investors can signal the value of a match with higher pay. In terms of assumptions, these studies differ from ours in that they do not model a moral hazard problem between a principal and an agent. In these models, the owner-manager is the residual claimant and, consistent with the signalling hypothesis, may retain excess exposure to the firm’s risk to signal quality. In terms of results, this literature is typically consistent with excess exposure while our model predicts that the opposite may sometimes be true.

A second set of papers, while focusing on signalling as well, are part of a large body of literature that deals with principal-agent problems in which the principal has more information on the production technology than the agent (the “informed-principal” paradigm). Melumad and Thoman (1990) discuss an informed principal model in which the firm is ex-ante informed about being compliant and hires a monitor (the auditor) who is initially uninformed and whose effort is subject to moral hazard. Chen, Hemmer and Zhang (2008) show that an informed board will signal better prospects to the agent by offering higher pay and using garbled performance measures. Baldenius and Meng (2008) discuss how entrepreneurs may signal their type to active investors who may contribute to the firm overall value, and describe how these distortions may affect equilibrium effort as a function of the riskiness of the environment. Our model is similar to these approaches in the sense that it includes both signalling motives and a moral-hazard problem. However, in these settings, the problem lies with differences of information about performance between the principal and the agent. By contrast, in our model, the principal and the agent (who are both corporate insiders) are assumed to be equally informed.
1. Institutional Background

Disclosure of compensation and all forms of stock-based ownership for firms whose ownership is made available to the public are subject to Regulation S-K as part of the Securities and Exchange Commission Act of 1934 and subsequent amendments. The oldest form of management incentive disclosure refers to disclosures of equity ownership, which traces its origins as far back as the original SEC Act of 1934; according to section 16, directors and officers are required to file a statement disclosing the amount of all securities owned, as well as changes to security ownership. The SEC provides three forms for such disclosures, Form 3 indicates new ownership of equity-based securities, Form 4 indicates changes in equity ownership and should be filed between 2 and 10 days after the transaction has taken place, Form 5 is an annual statement of changes in ownership. Such disclosures are not restricted to stock and incorporate other securities commonly used in compensation plans such as restricted equity, stock options or phantom shares. Each forms provides a statement of changes in holdings of a particular class of securities as the net amount of holdings after the transaction has taken place. These forms can be downloaded electronically from the SEC website between 1993 and 2008 (http://idea.sec.gov/) and are publicly available to outside investors. In theory, investors should be able to perfectly know managerial equity holdings from such forms; however, from a practical perspective, consolidating all such forms (which are filed for every security transaction) can be difficult and a table summarizing current ownership is generally given in corporate proxy statements.

Disclosure of managerial contract is regulated as part of paragraph 209.402 in Regulation S-K. The regulation requires “clear, concise and understandable disclosure of all plan and non-plan compensation awarded to, earned by, or paid to the named executive officers”. This includes, among other things, disclosures of salary, bonuses, pension contributions, the fair value of stock and option awards as well as the fair value of corporate benefits provided by the firm to the executive (as amended in September 8 2006). In addition, the regulation requires additional disclosures of “factors considered in decisions to increase or decrease compensation materially”. There are however several important differences between stock-based disclosures and compensation disclo-

\[2\] For an electronic version of the document, see http://www.law.uc.edu/CCL/34Act/sec16.html. In the current amended version, the rules for disclosure of security ownership are part of paragraph 209.403 in regulation S-K.

sures. First, such compensation disclosures are generally with regards to previously paid compensation and may be less informative on pay-for-performance for future periods. Second, no quantitative assessment of contingent performance payoff is required: “registrants are not required to disclose target levels with respect to specific quantitative or qualitative performance-related factors considered by the compensation committee or the board of directors”. These disclosures are typically disclosed as part of shareholder proxy statements, but may also be found in 10-K filings or annual reports provided by investor relations.

While these requirements are the minimum level of information admissible by the SEC, some firms voluntarily provide additional information to their shareholders. An interesting example is given in General Motors annual report, which provides a number of details about management incentives. The firm discloses that equity ownership by the CEO is set at a target 7 times base salary, and discloses weights associated to each performance measure that takes part in the bonuses. The compensation plan involves estimates for severance expenses broken down by terminations after mutually-acceptable performance and firings (in such cases long-term incentive plan payments are lost). Finally, the annual report provides additional information on target base salary decreases for the next year from 99% for the CEO to 10% for other officers for 2009. However, such detailed disclosures are the exception rather than the norm and correspond to the particular economic context and shareholder activism faced by some companies. In summary, most of the forward-looking information available to outside investors about pay-for-performance levels is contained in managerial equity ownership. In comparison, there is generally no equivalent ex-ante disclosure for other forms of compensation such as salary or bonuses.

2. The Model

We give a brief non-technical sketch of the model. There are five participants: (i) a set of initial shareholders (the principal), (ii) a manager (the agent), (iii) a privately-informed fundamental trader (the investor), (iv) a set of liquidity-motivated traders (the noise traders) and (v) a competitive market maker. There is a single period with two event dates, the contracting date and the trading date.

During the contracting date, the principal and the agent receive advance information about
the firm’s potential earnings. We model this information as a productivity signal about the return on the agent’s effort: specifically, more productive firms generate higher earnings per unit of effort (similar to Kydland and Prescott (1982)’s total factor productivity). Effort is unobservable to the principal so that, to resolve the moral hazard problem, the principal offers an incentive compensation arrangement. Given that only performance-pay due to equity ownership is, in practice, ex-ante known to outside investors, we restrict our attention to equity compensation. For reasons of tractability, we assume that the agent’s equity portfolio can be approximated as a linear claim (similar to several prior studies, e.g., Holmström and Tirole (1993) or Baldenius and Meng (2008)), as with regular stock or options that are well in the money.

During the trading date, the level of equity incentives is disclosed to outside investors; however, the information about the firm’s productivity cannot be credibly disclosed. For example, we have in mind soft information that is based on internal forecasts or complex business specific information that cannot be disclosed without knowledge of the operations of the firm. There exists a fundamental investor (e.g., a fund manager) who receives additional information about the firm’s earnings and can make a trade based on both the incentive plan disclosure and this fundamental signal. Conditional on the disclosure and the total order flow, a competitive market maker determines the market price for the firm’s equity.

In summary, our model is one in which contracting parties may have superior information and can write an incentive contract based on a public price. Information flows are two-way: one, from the firm to the capital market via the choice of a particular contract and what it may indicate on the firm’s prospects; and, two, from the capital market to the firm via the information contained in the stock price about the firm’s actual earnings and what it may reveal about the agent’s choice of effort. The formal details of the model are described in the next paragraphs.

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It should be noted that the information used to justify a high productivity may be proprietary (e.g., revealing the firm’s current patents or detailed data about operating segments), although the fact that the firm is high-productivity may not be. For example, a software manufacturer may not credibly disclose to outside investors the potential of a new platform without disclosing detailed information about the features that this platform will incorporate or the source code - however, doing so would make it possible for competitors to replicate these feature and thus would be detrimental to the firm’s competitive position (as in Verrecchia (1983)). On the other hand, there may be no loss in this firm’s competitive hedge if only the fact that the platform will be successful is disclosed by other means.
Nature draws firm’s type $\alpha$.  
$\alpha \in \{\alpha_h, \alpha_l\}$ with probability $p$ and $1-p$ resp. 
Principal and agent learn $\alpha$. 
Principal offers contract $w(P) = A + BP$ 
Agent accepts or rejects, and chooses effort $e$.  
Firm’s earnings are: $\hat{\pi} = \alpha e + \epsilon$ 
$B$ is publicly disclosed. 
Fundamental investor receives extra information $\theta = \hat{\pi} + \eta$ and posts trade $Y$ 
Market maker posts price $P = \mathbb{E}(\hat{\pi}|Y + u, B)$

Figure 1: Model Timeline

Contracting Date

The principal and the agent receive private information $\alpha$ about the firm’s productivity, which can be high $\alpha = \alpha_h$ with probability $p \in (0, 1)$ or low $\alpha = \alpha_l$ with probability $1 - p$, where $0 \leq \alpha_l < \alpha_h$. Denoting $e$ the agent’s effort, the firm’s earnings $\pi$ are given as follows:

$$\pi = \alpha e + \epsilon$$  
(2.1)

The random variable $\epsilon$ represents stochastic events that are not under the control of the agent and it is Normally distributed with mean zero and variance $\sigma^2_{\epsilon}$. One may interpret $\sigma_{\epsilon}$ as representing the amount of economic uncertainty such as, for example, the inherent volatility of a firm’s earnings. The parameter $\alpha$ represents the return on effort, and thus captures the firm’s potential profitability.\(^5\) In short-hand, we refer to this parameter as the firm’s “type.” All noise terms are assumed to be independent white noise. We assume that the principal can offer an equity contract in which the agent is compensated relative to the firm’s stock price $P$, i.e. $w(P) = A + BP$ where $A$ and $B$ are two parameters to be endogenously determined.\(^6\)

The agent’s preference is given by an exponential utility function with absolute risk-aversion coefficient $r$, and a cost of effort $e^2/2$.\(^7\) Based on these preferences, the agent’s utility function $U$

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\(^5\)The interpretation of the multiplicative form is that by high or low quality, we mean the sensitivity of the firm’s earnings to unobservable effort; specifically, our results are unchanged if there are other public factors affecting cash flows that are correlated to $\alpha$ (i.e., $\hat{\pi} = \alpha e + \epsilon + \epsilon'$ with $\epsilon'$ public). For example, a situation of interest may be the case of a firm that is restructuring; for such a firm, $\alpha$ may be very high simultaneously with a very negative value for $\epsilon'$. On the other hand, our results would be affected if outside investors knew neither $\alpha$ nor the extra factor $\epsilon'$ since signalling would now occur over a multi-dimensional signal.

\(^6\)We are only interested in the piece of the compensation that could be ex-ante observed by outside investors. In the vast majority of cases, this corresponds to equity compensation which can be inferred by equity ownership. Other forms of pay for performance, such as discretionary bonuses, are only observable ex-post and would not be useful as a reporting channel.

\(^7\)If effort were to be $ce^2/2$, then the $c$ could be normalized to $c' = 1$ and each $\alpha_i$ normalized to $\alpha'_i = \alpha_i/c$ ($i = h, l$).
can be written as follows:

\[ U = \mathbb{E}(w(P)) - \frac{r}{2} V(w(P)) - \frac{e^2}{2} \] (2.2)

**Trading Date**

For now, let us denote \( I \) the information available to both the fundamental investor and the market maker. The investor privately observes a signal \( \theta \) about the firm’s cash flow:

\[ \theta = \pi + \eta \] (2.3)

where \( \eta \) is Normally distributed with mean 0 and variance \( \sigma^2_\eta \).

Based on this information, the investor posts a demand \( Y \) for the firm’s stock. In addition, noise traders submit a demand \( u \) which is Normally distributed with mean 0 and variance \( \sigma^2_u \). As in Kyle (1985), a competitive market maker observes the aggregate order flow \( X = Y + u \) and posts a price for the asset. The competitive price is given by:

\[ P = \mathbb{E}(\tilde{\pi} | Y + u, I) \] (2.4)

As is common in the financial reporting literature, we assume that the principal has a short-term horizon (Verrecchia (1983), Dye (1985)); namely, assume that the principal chooses the contract \((A, B)\) to maximize the market price of the firm, minus the cost of the compensation.\(^8\)

\[ V = \mathbb{E}(P - w(P)|\alpha) \] (2.5)

### 3. Full Disclosure Benchmark

As a preliminary step to the analysis, we determine how the market price is determined as a function of actual and anticipated effort. Then, we derive the benchmark second-best solution

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\(^8\)The assumption that the compensation is paid by the principal is common in previously cited signalling papers and not essential for the intuitions at play. By making this assumption, we also want to avoid an unreasonable potential situation in which the principal can expose future shareholders to a large unexpected compensation liability.
to the model when the contract and the firm’s type are publicly observable. In the remaining Sections, we will use this benchmark as a reference point to examine the consequences on the optimal contract and firm value when the firm’s type is no longer observable to investors.

3.1. Price Discovery

We focus on equilibria in which the market maker follows a pricing rule that is linear in the order flow, i.e. $P(X)$ linear in $X$. We restrict the attention to fully-separating equilibria in which investors can, on the equilibrium path, perfectly infer the type of each firm.\(^9\) Let $\hat{\alpha}$ and $\hat{e}$ denote, respectively, the conjectured type and effort (we will set conjectures equal to actuals when such variables are observable). In the next Lemma, we characterize the trading decisions of the fundamental investor and the price set by the market maker. Let $\hat{\alpha}$ and $\hat{e}$ be, respectively, the type and the effort as anticipated by outside investors.\(^10\)

(i) The informed trader purchases $Y = d_0 + d_1 \theta$ shares of equity, where:

\[
\begin{align*}
  d_0 &= \hat{\alpha} \hat{e} \\
  d_1 &= \sigma_u \sqrt{\sigma_e^2 + \sigma_\eta^2}
\end{align*}
\]

(ii) The market maker offers a price $P(X) = \rho_0 + \rho_1 X$, where:

\[
\begin{align*}
  \rho_0 &= -\hat{\alpha} \hat{e} \frac{\sigma_u}{\sqrt{\sigma_e^2 + \sigma_\eta^2}} \\
  \rho_1 &= \frac{1}{2} \frac{\sigma_e^2}{\sigma_u \sqrt{\sigma_e^2 + \sigma_\eta^2}}
\end{align*}
\]

\(^9\)It should be noted that, in our model, the restriction to fully-separating equilibria is implied by the restriction to a linear equilibrium. If firms pooled in equilibrium, expectations conditional on a signal would no longer be linear in the signal (because the distribution of earnings in the pooled sample would be drawn from a mixture of Normal distributions); as a result, there would be no equilibrium in which prices are linear in the order flow. Further, even if we were to solve for non-linear pricing schedules, such non-linearities would make it extremely difficult to obtain the certainty equivalent of the agent (which would no longer take the form of a mean-variance expression).

\(^{10}\)One may note that we have implicitly restricted off-equilibrium beliefs to be identical for the market maker and the informed trader (this is to avoid equilibria sustained by off-equilibrium disagreement).
(iii) The market price process is given by

\[ P = m\hat{\alpha} + (1 - m)\alpha e + Z \]  

(3.5)

where \( m = \frac{\sigma^2 + 2\sigma_\eta^2}{2\sigma^2 + 2\sigma_\eta^2} \) and Z is a Normally distributed random variable with mean 0 and variance \( (1 - m)\sigma_e^2 \).

Lemma 3.1 states that the market price depends on equilibrium conjectures about effort and productivity. Actual earnings are partially reflected in prices due to the impact of the fundamental investor’s information on the total order flow. As shown in Equation (3.5), the parameter \( m \in (1/2, 1) \) represents the weight assigned to equilibrium conjectures and it is determined by two characteristics of the informational environment: (i) the information acquired by investors \( 1/\sigma_\eta \), (ii) the noise in firm earnings \( \sigma_e \). We explain next the dependence of \( m \) on these two variables. If the investor learns more from her signal (i.e., \( 1/\sigma_\eta \) high), she will trade on this information more aggressively, in turn leading the (price-protected) market-maker to increase the price sensitivity. As such, the market price will depend more, via the investor’s trade, on the actual choice of effort (i.e., \( m \) low). Following this same logic, the investor trades more aggressively when her information is more informative on actual earnings relative to the conjectured effort \( \hat{\alpha} \). Somewhat counter-intuitively, then, the more noisy the earnings (i.e., \( \sigma_e \) high), the more the investor trades and thus the more the market price responds to actual effort (i.e., \( m \) low). That is, contrary to common agency theory, the market may become a more useful contracting variable when earnings are less informative on effort. Indeed, if earnings are not noisy in effort (or, \( \sigma_e = 0 \)), then the market price \( P \) is no longer a function of actual effort and thus it will become impossible to elicit any positive effort \( e > 0 \) using only equity compensation.\(^{11}\)

Outside investors’ expectations about effort are given by \( \hat{\alpha} = (1 - m)\hat{B}\hat{\alpha} \), while the actual choice of effort is given by \( e = (1 - m)B\alpha \). To induce participation to the contract, the agent must be paid:

\[ A \geq \frac{r}{2} (1 - m)\sigma^2 B^2 - \frac{1}{2} (1 - m)^2 \alpha^2 B^2 - m(1 - m)\hat{\alpha}^2 B^2 \]  

(3.6)

\(^{11}\)Interestingly, this intriguing observation is partly related to the (relatively less known) second part of the original Dye (1985) article; Dye show that a more precise ex-ante disclosure can reduce the willingness of other parties to use their own information, leading to less overall information.
We are now equipped to analyze how information given to outside investors about the contract or productivity affects market prices and the choice of the optimal contract.

### 3.2. Known Productivity Benchmark

We solve next for the scenario in which both the firm’s type and the contract are observable to the investor, i.e. we set $\hat{\alpha} = \alpha$.\(^{12}\) We denote the solution to this problem $(A_{FI}^\alpha, B_{FI}^\alpha)$ and denote $V_{FI}^\alpha$ the corresponding firm surplus.

**Proposition 3.1** *In the Full Information regime,*

\[
A_{FI}^\alpha = \frac{1 - m \alpha^4 (r \sigma^2 - (1 + m) \alpha^2)}{2 (r \sigma^2 + (1 - m) \alpha^2)^2} \quad (3.7)
\]

\[
B_{FI}^\alpha = \frac{\alpha^2}{r \sigma^2 + (1 - m) \alpha^2} \quad (3.8)
\]

The principal achieves a surplus:

\[
V_{FI}^\alpha = \frac{\alpha^4 (1 - m)}{2 (r \sigma^2 + (1 - m) \alpha^2)} \quad (3.9)
\]

**Corollary 3.1** *Under full information, the incentive coefficient $B_{FI}^\alpha$ and the elicited effort are greater when the type $\alpha$ is higher.*

Corollary 3.1 establishes an important preliminary property of the model. For a greater type $\alpha$, the marginal benefit of inducing effort increases; in turn, the (second-best) optimal contract prescribes a higher pay-for-performance $B_{FI}^\alpha$ for a higher type. Importantly, in this model, pay-for-performance is associated to high future earnings but does not cause future earnings (since the pay-for-performance is chosen optimally and increasing $B$ beyond $B_{FI}^\alpha$ would have decreased ex-ante value). Another point worth noting here is that the fixed component of pay $A_{FI}^\alpha$ is ambiguous in the type $\alpha$. On the one hand, as noted earlier, a high type firm offers a greater level of pay-for-performance which, in turn, requires higher total pay. On the other hand, a high type firm will

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\(^{12}\)When quality is known, whether or not the full contract or only the equity component is observable is irrelevant. The component $A$ will always be set to bind the participation of the agent.
achieve a higher final stock price which, as a result, shifts a greater fraction of total pay toward equity compensation rather than fixed compensation.

4. Incomplete Information

4.1. Equilibrium Definitions

While useful as a benchmark, the Full-Information case requires restrictive assumptions about the information known to investors or how private information could be credibly conveyed to outsiders. Indeed, if private information is not observable, the payoff function of the principal and the agent makes it their best interest to over-report their type and increase the stock price.

In this respect, the association between productivity and the incentive scheme bears the following question: if the firm type is unobservable, can the compensation scheme be used as a credible signal? To address this question, we turn to the case in which quality is not observable and only the equity component \( B \) of the contract is observable to outside investors. That is, we incorporate into the model some information known to investors about the incentive scheme: specifically, managerial equity holdings are disclosed in regulatory SEC filings. On the other hand, firms provide little information about the level of salaries received in future periods (this tends to be disclosed ex-post). In addition, it may be difficult for outside investors to monitor ex-ante side payments to the manager, the exact value of perks and fringe benefits or the details of extra compensation based on other variables that are not necessarily well-observable to outsiders. To compare the results with other informational regimes, we will later on extend the results to another benchmark in which the principal discloses the entire contract \( (A, B) \).

A fully-separating equilibrium is defined as a Bayesian equilibrium of the game in which each type offers a different contract so that, conditional on observing which pay-for-performance sensitivity is chosen, outside investors can infer whether \( \alpha = \alpha_h \) or \( \alpha = \alpha_l \). In such an equilibrium, each type offers a contract of the form \( w(P) = A_{ED}^\alpha + B_{ED}^\alpha P \), where \( B_{ED}^{\alpha_h} \neq B_{ED}^{\alpha_l} \). We establish first that, as is common in signalling problems (Riley (1975)), a fully-separating equilibrium features no distortion to the contract chosen by the low-type principal.

Lemma 4.1 In any Fully-Separating equilibrium, \((A_{ED}^{\alpha_l}, B_{ED}^{\alpha_l}) = (A_{FI}^{\alpha_l}, B_{FI}^{\alpha_l})\).
Define the Least-Cost (LC) separating equilibrium as the equilibrium that maximizes the surplus of the high productivity type. This restriction is standard in signalling games and eliminates Pareto-dominated separating equilibria with excessive distortions to the contract.

4.2. When $\alpha_l = 0$

We determine next the optimal contract $(A^{\alpha_h}_{ED}, B^{\alpha_h}_{ED})$ for the high type principal. In order to tease out the main intuition in a simplified setting, we first focus on the simpler case in which $\alpha_l = 0$, so that the low type firm does not benefit from effort. For example, this situation would correspond to an environment in which outside investors are uncertain about whether or not the firm has an investment opportunity that would benefit from the agent’s actions; thus, if no investment is available, the firm’s profit would be pure noise $\tilde{\pi} = \epsilon$. Clearly, if $\alpha_l = 0$, the low type firm must achieve an equilibrium surplus equal to zero. To derive a fully-separating contract, it is necessary to find $(A^{\alpha_h}_{ED}, B^{\alpha_h}_{ED})$ such that the low type principal would make negative surplus when choosing to offer this contract and, vice-versa, the high type principal makes a higher surplus by offering the equilibrium contract than offering a contract with $B = 0$.

**Proposition 4.1** Suppose $\alpha_l = 0$. If $m \leq r\sigma^2/\alpha_h^2$, there exists a unique LC fully-separating equilibrium such that: $(A^{\alpha_h}_{ED}, B^{\alpha_h}_{ED}) = (A^{\alpha_l}_{FI}, B^{\alpha_l}_{FI})$, and

\[
A^{\alpha_h}_{ED} = \frac{2\alpha_h^4(1-m)m^2(r\sigma^2_h - \alpha_h^2(1+m))}{r^2\sigma^2_\epsilon} \quad (4.1)
\]

\[
B^{\alpha_h}_{ED} = \frac{2m\alpha_h^2}{r\sigma^2_\epsilon} \quad (4.2)
\]

The high type principal achieves a surplus $V^{\alpha_h}_{ED}$ given by:

\[
V^{\alpha_h}_{ED} = \frac{2\alpha_h^4 m(1-m)(r\sigma^2_h - m\alpha_h^2)}{r^2\sigma^2_\epsilon} \quad (4.3)
\]

If $m > r\sigma^2_\epsilon/\alpha_h^2$, there exists no Fully-Separating equilibrium.

**Corollary 4.1** $B^{\alpha_h}_{ED} > B^{\alpha_l}_{FI}$ and $A^{\alpha_h}_{ED} > A^{\alpha_l}_{FI}$.

\(^{13}\)This comparison does not hold only for the LC fully-separating equilibrium, but for any fully-separating equilibrium (not necessarily LC). Specifically, the set of fully-separating equilibria of the game is the LC equilibrium as well as other equilibria with the incentive coefficient for the high type is even greater, beyond what is required to achieve separation.
The use of the contract as a signal of the firm’s type requires a distortion to the optimal contract. Specifically, when $\alpha_l = 0$, the high type principal offers more pronounced incentives (and higher pay) than in the full-information environment. To see why this is the case, consider the low type principal’s incentive to replicate the contract offered by the high type principal. Since $\alpha_l = 0$, when the low type offers $B_{ED}^{\alpha_h}$, there will be no effect on the agent’s effort or, as a result, on the fundamental signal received by the investor. It follows that the trade of the fundamental investor, on average, reduces the market price of this firm relative to the unconditional expected market price of a high type firm. This first effect explains why the marginal (productive) benefit of increasing pay-for-performance tends to be lower for the low-type firm and, thus, why a fully-separating contract must necessarily induce pronounced incentives as compared to the full-information environment.\(^{14}\)

There is an additional economic force that may cause the existence of a fully-separating equilibrium to fail. This property is unusual in standard signaling models and specific to several interesting interactions between incentive contracting and financial reporting. Note that the equity component of the agent’s pay is determined by the market price, which is itself a function of investors’ expectations. As a result, by increasing pay-for-performance, the contract also increases the sensitivity of the contract payments to private information. Thus, the increase in pay-for-performance incidentally increases the benefit of the low type principal to replicate the contract offered by the high type principal. When the market price is predominantly determined by market expectations and not the fundamental investors’ information (i.e., $m$ high), this reporting benefit becomes so large that separation becomes impossible.

4.3. When $\alpha_l > 0$

We turn next to the case of a positive productivity $\alpha_l > 0$. The main additional difficulty of this situation is due to the fact that, conditional on the low-productivity contract, the agent would now exercise some effort. The next Proposition presents the form of fully-separating equilibria in this situation.

\(^{14}\)As is usual in signalling games, it can be seen from the proof of the result that the incentive-compatibility condition for the low $\alpha$ principal binds while the incentive-compatibility condition for the high $\alpha$ principal does not bind. This is intuitive here as, all other things being equal, the high $\alpha$ principal will generate a higher expected market price.
Proposition 4.2 There exists $\alpha_1$ and $\alpha_2$ such that:

(i) If $m \geq r\sigma^2_t/\alpha^2_h$ and $\alpha_1 > \alpha_2$, in the LC fully-separating equilibrium, incentives are muted and given by:

$$B^\alpha_{ED} = \frac{m\alpha^2_h + (1-m)\alpha^2_l - \sqrt{m(\alpha^2_h - \alpha^2_l)(m\alpha^2_h + (2-m)\alpha^2_l)}}{r\sigma^2_t + (1-m)\alpha^2_l} < B^\alpha_{FI}$$

(ii) If $m < r\sigma^2_t/\alpha^2_h$ and $\alpha_1 \notin (\alpha_1, \alpha_2)$, in the LC fully-separating equilibrium, incentives are pronounced and given by:

$$B^\alpha_{ED} = \frac{m\alpha^2_h + (1-m)\alpha^2_l + \sqrt{m(\alpha^2_h - \alpha^2_l)(m\alpha^2_h + (2-m)\alpha^2_l)}}{r\sigma^2_t + (1-m)\alpha^2_l} > B^\alpha_{FI}$$

(iii) Otherwise, a fully-separating equilibrium does not exist.

We develop next a simple intuition for this extended characterization. When markets respond more to actual effort (i.e., low $m$), pronounced incentives tighten the low-type agent incentive compatibility, because market prices are more sensitive to realized earnings. Hence, the stronger the incentives, the more difficult it is for the low-type to mimic the earnings generated by the high type. In turn, as noted earlier, the case of information-sensitive market prices suggest a greater use of equity compensation.

As $m$ becomes larger, by contrast, the market reacts less to the privately-observed signals and more to conjectures. That, in turn, makes it more desirable for the low type to mimic the high type so as to realize the higher price. Indeed, when $m > r\sigma^2_t/\alpha^2_h$ (i.e., the threshold on $m$ to guarantee the existence of a fully-separating equilibrium), separation cannot be used through higher incentive pay but rather by reducing the total surplus achieved by the high type principal. In this case, the fully-separating equilibrium features muted incentives as compared to the full-information environment.\(^\text{16}\)

The model also implies that the distortion to the incentive coefficient can be tied to characteristics of managerial preferences and market characteristics. Specifically, incentives are pronounced

\(^{15}\)The expressions for these parameters are given in the Appendix and are quite cumbersome; we also stress that, when $m < r\sigma^2_t/\alpha^2_h$, the region of parameter values such that a fully-separating does not exist may be empty.

\(^{16}\)I thank an anonymous reviewer for suggesting this intuition for the result.
when the market is more responsive ($m$ low), when the agent is more risk-averse ($r$ high), when earnings are more noisy ($\sigma^2 \epsilon$ high) and when high effort is less valuable ($\alpha_h$ high).

One additional economic intuition for these results can be tied to several prior results on signaling and agency theory. In a pure signaling environment (e.g., Sapra (2002), Arya and Mittendorf (2005)), the informed party should retain some residual ownership to credibly convey information to the outside party. If the moral hazard problem would, absent signaling motives, prescribe low levels of residual ownership, then the signalling motives will increase the level of pay-for-performance. In this respect, the signalling motive increases the (ex-ante insufficient) level of residual ownership present in the moral hazard problem. Consistent with this interpretation, we observe that factors that decrease the level of incentive pay in the full-information (such as high risk-aversion or more noisy earnings) tend to lead to more pronounced incentives when signaling is incorporated in the model. By contrast, when the moral hazard would prescribe very high incentives, these same incentives can create an excessive tension leading the low-type to mimic the high-type. As a result, the principal chooses muted incentives that lead to lower pay and effort than would have been optimal under pure moral hazard.

Finally, and specific to model, we find that, in certain environments, separation can be impossible. This occurs in two possible situations. First, if the information available to investors is low and the value of the low-quality firm is low as well, incentives for the low-quality firm to mimic are so strong that separation would require to set a performance pay that is so low that it would destroy most of the high-quality firm value. This would more than overcome the separation benefits. Second, if the information available to investors is high, separation involves a higher performance pay. If the low-quality firm has very low quality, then this extra effort is sufficient to provide the information required for separation. If the low-quality has very high quality, the low-quality firm is sufficiently well-off not to strongly desire mimicking. Failure of separation occurs for moderate low-quality firm since defeating incentives to mimic would require a prohibitively high performance pay.\footnote{Unfortunately, one limitation of our model is that one cannot characterize more explicitly behavior over parameter values such that separation occurs; this is because pooling equilibria would be inconsistent with the pricing assumption in our Kyle setting (since outsiders would expect firm values to be a mix of Normal distributions) and could not support a linear trading equilibrium.}
4.4. Full Contract Disclosure

Recent disclosure requirements have moved toward ex-ante disclosure over a larger scope of items representing some form of compensation. In particular, SEC requirements have changed in 2006 to incorporate, among other things, additional disclosures on the value of fringe benefits as well as disclosures on non-monetary retirement benefits. Such requirements are intended to make, if they are successful, a more complete managerial contract observable to outside investors; more precisely, it has added disclosure requirements on non-contingent components of compensation. It should be noted that, to this date, such disclosures remain backward-looking. However, to the extent that firms do not make important rapid changes to their managerial pay structure, such disclosures could be indicative of future pay.

We examine the most favorable case in which the entire compensation is observable to outside investors. This case should be regarded here as an extreme situation, which would be unlikely to be achieved in practice. However, by comparing the case with both \( A \) and \( B \) are observable with the case in which only \( B \) is observable, one may get a better idea of the consequences of increased disclosure requirements and how the legislation would be likely to affect management contracts. We let \( (A_{FD}, B_{FD})_{\alpha \in \{\alpha_h, \alpha_l\}} \) indicate the LC fully-separating contract in this alternative disclosure environment.

**Proposition 4.3** \( A_{FD}^\alpha = A_{FI}^\alpha \) and \( B_{FD}^\alpha = B_{FI}^\alpha \) for all \( \alpha \).

Proposition 4.3 points to the desirable signalling value of a complete managerial contract disclosure. Unlike with partial observability, full-separation between each productivity type is achieved with no distortion to incentives. In other words, it is incentive-compatible for each productivity type to choose his own full-information optimal contract, even though the productivity is not observable and the principal and the agent of the low-productivity firm would be better-off, all other things being equal, claiming to be a high productivity firm.

To understand why costless separation obtains, we need to discuss separately the incentive of each type not to mimic the contract offered to the other type. First, when observing a low-quality firm offering a high-quality contract, the agent would expect lower payoffs and thus would not participate to the contract. Thus, the agent’s outside option acts as a monitoring device that indicates
a contract deviation by the low-productivity principal. Second, the high-productivity firm may benefit from offering the low-productivity contract because of the lower compensation expense that such a contract would entail. However, the high-quality firm can only reduce compensation expense up to $\alpha_l F_l$ which creates a rent to the agent (recall that the high-quality firm, even after a deviation, generates on average higher market prices that the low-productivity firm). In addition to this opportunity cost, outside investors reduce the market price after such a deviation. As a result of these two forces, it is never optimal for the high-quality firm to offer the low-productivity contract.

5. Concluding Remarks

This paper develops a novel mechanism through which a principal may signal a firm’s type to outside investors. In our model, the principal does not need to retain any of the firm’s equity (unlike standard signalling models) but may competitively contract with a manager who is informed and may or may not provide effort. We show that the choice of effort is affected by both the level of performance-pay chosen by the principal and the quality of the firm. If contracts convey information on the firm, then our analysis shows how and why a firm’s stock price and future operating performance should be associated to the choice of a particular pay package. In this respect, the model offers a framework to tie firm performance and contracting choices, in an optimal contract setting.

We further develop several predictions for future empirical applications. Even if information about a firm’s type is known to market participants, we show that pay-for-performance will be positively associated to the firm’s type and thus, also, to the agent’s effort and future earnings. In the model, the observed level of pay-for-performance is endogenous and the positive association of performance with incentive pay does not imply that greater firm value could be achieved by further increasing pay-for-performance. Examining reporting motives, we show that concerns for higher market prices can distort the optimal contract: namely, we show that when markets are more (less) responsive to information, such reporting motives lead to pronounced (muted) equity ownership, and higher (lower) pay and managerial effort. From a policy perspective, in these cases, mandated changes to compensation arrangements or their disclosure may potentially increase the efficiency
of contracts.

Appendix

Proof of Lemma 3.1: Let $P(X) = \rho_0 + \rho_1 X$ denote the conjectured price offered by the market maker as a function of the order flow $X$. The surplus of the informed trader is:\(^{18}\)

$$E(Y(\pi - P)|\theta, Y) = Y(E(\pi|\theta) - \rho_0 - \rho_1 Y)$$

The optimal choice of $Y$ is:

$$Y^* = \frac{1}{2\rho_1} \frac{\sigma_n^2 \theta + \sigma_n^2 \hat{\alpha} \hat{e}}{\sigma_i^2 + \sigma_n^2} - \rho_0\text{ (5.1)}$$

It follows from the above that:

$$d_1 \rho_1 = \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2}\text{ (5.2)}$$

The market maker chooses a price that satisfies:

$$\rho_0 + \rho_1 X = E(\pi|X).$$

Note that $X = d_0 + d_1 \pi + d_1 \eta + u$ and therefore $(X - d_0)/d_1 = \pi + \eta + u/d_1$, where $\mathcal{V}(\eta + u/d_1) = \sigma_i^2 + \sigma_n^2/d_1^2$. It follows that the market pricing Equation for the market maker can be written:

$$\rho_0 + \rho_1 X = \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{\sigma_n^2 + \sigma_u^2/d_1^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} \frac{d_1 \sigma_n^2}{d_0} + \hat{\alpha} \hat{e}$$

$$= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} + \sigma_n^2 + \sigma_u^2/d_1^2 \hat{\alpha} \hat{e}$$

$$= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} + \sigma_n^2 + \sigma_u^2/d_1^2 \hat{\alpha} \hat{e}$$

$$= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} + \sigma_n^2 + \sigma_u^2/d_1^2 \hat{\alpha} \hat{e}$$

$$= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} + \sigma_n^2 + \sigma_u^2/d_1^2 \hat{\alpha} \hat{e}$$

$$= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_n^2 + \sigma_u^2/d_1^2} X - d_0 + \frac{d_1 \sigma_n^2}{d_0} + \frac{d_1 \sigma_n^2}{d_0} + \sigma_n^2 + \sigma_u^2/d_1^2 \hat{\alpha} \hat{e}$$

Identifying coefficients,

$$\rho_1 = \frac{d_1 \sigma_n^2}{d_0^2 \sigma_i^2 + \sigma_n^2 + \sigma_u^2}$$

$$d_1 \rho_1 = \frac{d_1 \sigma_n^2}{d_0^2 \sigma_i^2 + \sigma_n^2 + \sigma_u^2}$$

\(^{18}\)To save on notations, we have omitted the dependence of each of these expectations on the public information available to investors $I$.
From Equations (5.2) and (5.4), it follows that:

\[
\frac{1}{2} \sigma_u^2 = \frac{d_1^2 \sigma_u^2}{d_1^2 (\sigma_u^2 + \sigma_n^2) + \sigma_n^2} \\
2(\sigma_u^2 + \sigma_n^2) d_1^2 \sigma_u^2 = d_1^2 (\sigma_u^2 + \sigma_n^2) \sigma_u^2 + \sigma_n^2 \sigma_u^2 \\
\sigma_u^2 \sigma_u^2 = d_1^2 (2(\sigma_u^2 + \sigma_n^2) \sigma_u^2 - (\sigma_u^2 + \sigma_n^2) \sigma_u^2) \\
d_1 = \frac{\sigma_u}{\sqrt{\sigma_u^2 + \sigma_n^2}} \tag{5.5}
\]

From Equation (5.2),

\[
\rho_1 = \frac{1}{2} \frac{\sigma_u^2}{\sigma_u^2 + \sigma_n^2} \sqrt{\sigma_u^2 + \sigma_n^2} = \frac{1}{2} \frac{\sigma_u^2}{\sigma_u \sqrt{\sigma_u^2 + \sigma_n^2}}
\]

Next, one can obtain from Equation (5.1) that:

\[
d_0 = \frac{1}{2 \rho_1} \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_n^2} - \rho_0 \right) = \frac{\sigma_u \sqrt{\sigma_u^2 + \sigma_n^2}}{\sigma_u^2 + \sigma_n^2} \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_n^2} - \rho_0 \right) \\
= \frac{\sigma_u \sigma_n^2}{\sigma_u^2 + \sigma_n^2} \hat{\rho} - \rho_0 \frac{\sigma_u \sqrt{\sigma_u^2 + \sigma_n^2}}{\sigma_u^2 + \sigma_n^2} \tag{5.6}
\]

Next, it follows from Equation (5.3),

\[
\rho_0 = \frac{\hat{\sigma} \varepsilon (\sigma_u^2 + d_1^2 \sigma_u^2) - \hat{d}_0 d_1 \sigma_u^2}{d_1^2 (\sigma_u^2 + \sigma_n^2) + \sigma_n^2} \\
\hat{\sigma} \varepsilon (\sigma_u^2 + \sigma_n^2 + \sigma_n^2) \sigma_u^2 - d_0 \frac{\hat{\sigma} \varepsilon \sigma_u^2}{\sqrt{\sigma_u^2 + \sigma_n^2}} \\
= \frac{\hat{\sigma} \varepsilon (\sigma_u^2 + \sigma_n^2 + \sigma_n^2) \sigma_u^2}{\sigma_u^2 + \sigma_n^2} - d_0 \frac{\sigma_u}{\sqrt{\sigma_u^2 + \sigma_n^2}} \sigma_u^2 \\
= \frac{\hat{\sigma} \varepsilon (\sigma_u^2 + \sigma_n^2 + \sigma_n^2) - d_0}{\sigma_u^2 + \sigma_n^2} \\
= \frac{\hat{\sigma} \varepsilon (\sigma_u^2 + \sigma_n^2)}{\sigma_u^2 + \sigma_n^2} - d_0 \frac{\sigma_u^2}{\sigma_u^2 \sqrt{\sigma_u^2 + \sigma_n^2} + \sigma_u^2} \\
= \frac{\hat{\sigma} \varepsilon (\sigma_u^2 + \sigma_n^2)}{2(\sigma_u^2 + \sigma_n^2)} - \frac{\sigma_u \sigma_n^2}{2(\sigma_u^2 + \sigma_n^2)} \frac{\hat{\sigma} \varepsilon}{\sigma_u^2 + \sigma_n^2} \\
= \frac{\sigma_u \sigma_n^2}{2(\sigma_u^2 + \sigma_n^2)} \frac{\hat{\sigma} \varepsilon + \frac{\sigma_u}{2} \rho_0}{2} = \frac{\hat{\sigma} \varepsilon + \frac{\sigma_u}{2} \rho_0}{2} = \hat{\sigma} \varepsilon - \frac{\sigma_u}{\sqrt{\sigma_u^2 + \sigma_n^2}}
\]

Substituting $\rho_0$ from Equation (5.6),

\[
d_0 = \frac{\sigma_u \sigma_n^2}{\sqrt{\sigma_u^2 + \sigma_n^2}} \hat{\sigma} \varepsilon - \frac{\sigma_u \sqrt{\sigma_u^2 + \sigma_n^2}}{\sigma_u^2 + \sigma_n^2} \hat{\sigma} \varepsilon = \frac{\hat{\sigma} \varepsilon - \sigma_u}{\sqrt{\sigma_u^2 + \sigma_n^2}} \sigma_u
\]
Finally, in the market pricing function,

\[ P = \rho_0 + \rho_1 X \]

\[ = \rho_0 + \rho_1 d_0 + \rho_1 d_1 \alpha + \rho_1 d_1 (\epsilon + \eta) + \rho_1 u \]

\[ = \hat{\alpha} \epsilon - \frac{1}{2} \frac{\sigma_u^2}{\sigma^2 + \sigma_u^2} \hat{\alpha} \epsilon + \frac{1}{2} \frac{\sigma_u^2}{\sqrt{\sigma^2 + \sigma_u^2}} \sqrt{\sigma^2 + \sigma_u^2} (\alpha \epsilon + \epsilon + \eta) + \frac{1}{2} \frac{\sigma_u^2}{\sqrt{\sigma^2 + \sigma_u^2}} u \]

\[ = \frac{\sigma^2}{2(\sigma^2 + \sigma_u^2)} \hat{\alpha} \epsilon + \frac{\sigma^2}{2(\sigma^2 + \sigma_u^2)} \alpha \epsilon + \frac{\sigma^2}{2(\sigma^2 + \sigma_u^2)} (\eta + \epsilon) + \frac{1}{2} \frac{\sigma_u^2}{\sqrt{\sigma^2 + \sigma_u^2}} u \]  

(5.7)

\[ Z \] is a mean-zero Normally-distributed random variable with variance:

\[ V(Z) = \frac{1}{4} \frac{\sigma_u^4}{(\sigma^2 + \sigma_u^2)^2}(\sigma^2 + \sigma_u^2) + \frac{1}{4} \frac{\sigma_u^4}{\sigma^2(\sigma^2 + \sigma_u^2)} \sigma_u^2 = \sigma_v^2(1 - m) \]

\[ \square \]

**Proof of Lemma 3.1:** Given the price process in Equation (5.7), the expected utility of the agent is:

\[ U = \mathbb{E}(A + BP) - \frac{r}{2} V(A + BP) - \frac{e^2}{2} \]

\[ = A + B \hat{m} \hat{\alpha} + B (1 - m) \alpha - \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{e^2}{2} \]

This problem is concave in \( \epsilon \) with

\[ \frac{\partial U}{\partial \epsilon} = B(1 - m) \alpha - \epsilon \]

And therefore: \( \hat{\epsilon} = B(1 - m) \hat{\alpha} \). Finally, the agent must receive \( U \) greater than zero.

\[ 0 \leq U \]

\[ \leq A + B \hat{m} \hat{\alpha} + B (1 - m) \alpha - \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{e^2}{2} \]

\[ \leq A + m \alpha^2 B^2 (1 - m) + B (1 - m)^2 B \alpha^2 - \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{1}{2} B^2 (1 - m)^2 \alpha^2 \]

Therefore:

\[ A \geq \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{1}{2} B^2 (1 - m)^2 \alpha^2 - m(1 - m) \hat{\alpha}^2 B^2 \]

\[ \square \]

**Proof of Proposition 3.1:** Under the assumption of Full Information, the wage paid to the agent is given by:

(Lemma (3.1))

\[ A = \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{1}{2} B^2 (1 - m)^2 \alpha^2 - m(1 - m) \alpha^2 B^2 \]

\[ = \frac{r}{2} (1 - m) \sigma_u^2 B^2 - \frac{1}{2} (1 - m)(1 + m) \alpha^2 B^2 \]

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The problem of the principal can then be written as:

\[ V = (1 - B)\alpha A - (1 - B)\alpha^2(1 - m)B - \frac{r}{2}(1 - m)\sigma^2B^2 + \frac{1}{2}(1 - m)(1 + m)\alpha^2B^2 \]  

(5.8)

This problem is concave in \( B \), therefore:

\[
\frac{\partial V}{\partial B} = (1 - 2B)\alpha^2(1 - m) - r(1 - m)\sigma^2B + (1 - m)(1 + m)\alpha^2B
\]

\[ = \alpha^2(1 - m) - B(2\alpha^2(1 - m) - r(1 - m)\sigma^2 - (1 - m)(1 + m)\alpha^2) \]

\[ = \alpha^2(1 - m) - B(\alpha^2(1 - m)^2 + r(1 - m)\sigma^2) \]

Therefore:

\[ B_{FI} = \frac{\alpha^2}{r\sigma^2 + (1 - m)\alpha^2} \]

\[ A_{FI} = B^2\frac{1}{2}(1 - m)(r\sigma^2 - (1 + m)\alpha^2) \]

\[ = \frac{1 - m}{2} \frac{4\alpha^4(r\sigma^2 - (1 + m)\alpha^2)}{(r\sigma^2 + (1 - m)\alpha^2)^2} \]

Using these expressions to obtain the surplus of the principal:

\[ V = (1 - B_{FI})\alpha^2B_{FI}(1 - m) - A_{FI} \]

\[ = \alpha^2 B_{FI}(1 - m) - B^2_{FI}(1 - m)(\alpha^2 + \frac{r\sigma^2}{2} - (1 + m)\alpha^2) \]

\[ = \frac{4\alpha^4(1 - m)}{r\sigma^2 + (1 - m)\alpha^2} - \frac{4\alpha^4}{(r\sigma^2 + (1 - m)\alpha^2)^2}(1 - m)\sigma^2 - (1 - m)\alpha^2 \]

\[ = \frac{4\alpha^4(1 - m)}{2(r\sigma^2 + (1 - m)\alpha^2)^2} \]

To prove the Corollary, note that:

\[ \frac{\partial B_{FI}}{\partial \alpha^2} = \frac{(r\sigma^2 + (1 - m)\alpha^2) - \alpha^2(1 - m)}{(r\sigma^2 + (1 - m)\alpha^2)^2} = \frac{r\sigma^2}{(r\sigma^2 + (1 - m)\alpha^2)^2} > 0 \]

\( \square \)

**Proof of Lemma 4.1:** Suppose \( (A_{ED}^{\alpha_i}, B_{ED}^{\alpha_i}) \neq (A_{FI}^{\alpha_i}, B_{FI}^{\alpha_i}) \) and let \( V' \) denote the surplus of the principal under contract \( (A_{ED}^{\alpha_i}, B_{ED}^{\alpha_i}) \). Suppose the low-type firm deviates to offer the off-equilibrium contract \( (A_{FI}^{\alpha_i}, B_{FI}^{\alpha_i}) \).

Case 1: Outside investors believe that the firm is low type with probability one. Then, the agent would participate to the contract and the principal would achieve a surplus \( V_{FI} > V' \), which implies by construction greater surplus than any other contract; a contradiction to \( (A_{ED}^{\alpha_i}, B_{ED}^{\alpha_i}) \) being offered.

Case 2: Outside investors believe that the firm is high-type with probability one. Let \( V'' \) be the surplus of the
principal following this deviation.

\[ V'' = (1 - B_{FI}^\alpha)(m\hat{e} + (1 - m)ae) - A_{FI}^\alpha \]

\[ = (1 - B_{FI}^\alpha)B_{FI}^\alpha(m\hat{a}_h + (1 - m)\alpha_h^2) - A_{FI}^\alpha \]

\[ > (1 - B_{FI}^\alpha)B_{FI}^\alpha\alpha_h^2 - A_{FI}^\alpha = V' \]

Therefore, the low productivity would deviate to offer the Full Information contract.

**Proof of Proposition 4.1:** Let \( V_{ED}^{\alpha_h} \) be the surplus achieved by the high-type principal. From Equation (5.8),

\[ V_{ED}^{\alpha_h} = (1 - B_{ED}^{\alpha_h})\alpha_h^2(1 - m)B_{ED}^{\alpha_h} - \frac{r}{2}(1 - m)\sigma_h^2(B_{ED}^{\alpha_h})^2 + \frac{1}{2}(1 - m)(1 + m)\alpha_h^2(B_{ED}^{\alpha_h})^2 \]

\[ = B_{ED}^{\alpha_h}\alpha_h^2(1 - m) - (B_{ED}^{\alpha_h})^2(\alpha_h^2(1 - m) + \frac{r}{2}(1 - m)\sigma_h^2 - \frac{1}{2}(1 - m)(1 + m)\alpha_h^2) \]

\[ = B_{ED}^{\alpha_h}\alpha_h^2(1 - m) - \frac{1}{2}(1 - m)(B_{ED}^{\alpha_h})^2((1 - m)\alpha_h^2 + r\sigma_h^2) \quad (5.9) \]

We analyze next each incentive-compatibility constraint: (i) the low-type principal does not deviate to offer the high-type contract, (ii) the high-type principal does not deviate to offer the low-type contract.

(i) Suppose the low-type principal offers the high-type contract. Let \( \tilde{A} \) be the fixed transfer that needs to be paid to the agent. By Lemma 3.1,

\[ \tilde{A} = \frac{r}{2}(1 - m)\sigma_h^2(B_{ED}^{\alpha_h})^2 - m(1 - m)\alpha_h^2(B_{ED}^{\alpha_h})^2 \]

Writing the incentive-compatibility condition for the low type,

\[ 0 \geq (1 - B_{ED}^{\alpha_h})(m(1 - m)\alpha_h^2 B_{ED}^{\alpha_h}) - \tilde{A} \]

\[ \geq m(1 - m)\alpha_h^2 B_{ED}^{\alpha_h} - \left( \frac{r}{2}(1 - m)\sigma_h^2 - m(1 - m)\alpha_h^2 + m(1 - m)\alpha_h^2(B_{ED}^{\alpha_h})^2 \right) \]

\[ \geq m\alpha_h^2 B_{ED}^{\alpha_h} - \frac{r}{2}\sigma_h^2(B_{ED}^{\alpha_h})^2 \]

For this inequality to be satisfied, it must hold that \( B_{ED}^{\alpha_h} \notin (0, B_{FI}^\alpha) \) where:

\[ \overline{B}_{FI} = \frac{2m\alpha_h^2}{r\sigma_h^2} \geq \frac{\alpha_h^2}{r\sigma_h^2 + (1 - m)\alpha_h^2} = B_{FI}^\alpha \]

(ii) We verify next the incentive-compatibility condition for the high-type principal. Note that, when offering a contract with incentive coefficient equal to zero, the high-type principal would need to make a fixed payment \( A \) equal
to zero as well (the agent does not bear any risk and does not provide effort). Then, one can obtain:

\[ V_{ED}^{\alpha_h} = B_{ED}^{\alpha_h} \alpha_h^2 (1 - m) - \frac{1}{2} (1 - m) (B_{ED}^{\alpha_h})^2 ((1 - m) \alpha_h^2 + r \sigma_r^2) \]

\[ \leq m \alpha_h^2 B_{ED}^{\alpha_h} - \frac{r}{2} \sigma_r^2 (B_{ED}^{\alpha_h})^2 + B_{ED}^{\alpha_h} \alpha_h^2 (1 - 2m) - \frac{1}{2} (1 - m)^2 (B_{ED}^{\alpha_h})^2 \alpha_h^2 \]

\[ \leq B_{ED}^{\alpha_h} \alpha_h^2 (1 - 2m) - \frac{1}{2} (1 - m)^2 (B_{ED}^{\alpha_h})^2 \alpha_h^2 \quad \text{(IC for low type)} \]

\[ < 0 \quad \text{(m greater than 1/2)} \]

the incentive-compatibility condition can be written as follows:

\[ 0 \leq V_{ED}^{\alpha_h} \]

\[ \leq B_{ED}^{\alpha_h} \alpha_h^2 (1 - m) - \frac{1}{2} (1 - m) (B_{ED}^{\alpha_h})^2 ((1 - m) \alpha_h^2 + r \sigma_r^2) \]

\[ \leq m \alpha_h^2 B_{ED}^{\alpha_h} - \frac{r}{2} \sigma_r^2 (B_{ED}^{\alpha_h})^2 + B_{ED}^{\alpha_h} \alpha_h^2 (1 - 2m) - \frac{1}{2} (1 - m)^2 (B_{ED}^{\alpha_h})^2 \alpha_h^2 \]

\[ \leq B_{ED}^{\alpha_h} \alpha_h^2 (1 - 2m) - \frac{1}{2} (1 - m)^2 (B_{ED}^{\alpha_h})^2 \alpha_h^2 \quad \text{(IC for low type)} \]

\[ < 0 \]

When deviating to offering the contract of the low-type, the high-type agent makes zero profit (zero effort), and therefore the IC can be written \( V_{ED}^{\alpha_h} \) and is necessarily satisfied when that of the low-type is satisfied. To conclude, note that the LC-separating contract prescribes setting the incentive-compatible coefficient \( B_{ED}^{\alpha_h} \) closest to the Full-Information contract, and thus prescribes \( B_{ED}^{\alpha_h} = \overline{B}_l \). Then, by Lemma 3.1,

\[ A_{ED}^{\alpha_h} = \frac{r}{2} (1 - m) \sigma_r^2 \overline{B}_l^2 - \frac{1}{2} \overline{B}_l^2 (1 - m) \alpha_h^2 - m (1 - m) \alpha_h^2 \overline{B}_l^2 \]

\[ = (1 - m) m^2 \frac{2 \alpha_h^4}{r \sigma_r^2} - \alpha_h^2 (1 - m) (1 + m) \frac{2 m \alpha_h^2}{r^2 \sigma_r^2} \]

\[ = 2 \alpha_h^4 \frac{(1 - m) m^2 (r \sigma_r^2 - \alpha_h^2 (1 + m))}{r^2 \sigma_r^2} \]

and:

\[ V_{ED}^{\alpha_h} = \overline{B}_l \alpha_h^2 (1 - m) - \frac{1}{2} (1 - m) \overline{B}_l^2 ((1 - m) \alpha_h^2 + r \sigma_r^2) \]

\[ = \frac{2 m (1 - m) \alpha_h^4}{r \sigma_r^2} - \frac{2 m^2 (1 - m) \alpha_h^4}{r^2 \sigma_r^2} ((1 - m) \alpha_h^2 + r \sigma_r^2) \]

\[ = \frac{2 \alpha_h^4 m (1 - m)^2 (r \sigma_r^2 - m \alpha_h^2)}{r^2 \sigma_r^2} \]

It follows that a LC fully-separating equilibrium exists if and only if the above expression is positive, i.e. if \( m \leq r \sigma_r^2 / \alpha_h^2 \). □

**Proof of Proposition 4.2:** To establish that \( B_{ED}^{\alpha_h} \) implies a fully-separating equilibrium, we need to verify that: 1. the low-type principal would not deviate to offer the contract \( (A_{ED}^{\alpha_h}, B_{ED}^{\alpha_h}) \), 2. the high-type principal would not deviate to offer the contract \( (A_{EF}^{\alpha_l}, B_{EF}^{\alpha_l}) \) (recall by Lemma 4.1 that the low-type principal must choose the Full-Information
1. Suppose the low-type principal deviates to offer \((A_{ED}^{\alpha h}, B_{ED}^{\alpha h})\). Then, by Lemma 3.1, the fixed compensation offered by the low-type principal \(\hat{A}\) must satisfy:

\[
\hat{A} = \frac{r}{2}(1-m)\sigma_l^2(B_{ED}^{\alpha h})^2 - \frac{1}{2}(B_{ED}^{\alpha h})^2(1-m)^2\alpha_l^2 - m(1-m)\alpha_h^2(B_{ED}^{\alpha h})^2
\]

Let \(V_l\) denote the surplus of the low-type principal after this deviation has occurred.

\[
V_l = (1-B_{ED}^{\alpha h})(m\epsilon_l + (1-m)\sigma_l) - \hat{A}
\]

\[
= (1-B_{ED}^{\alpha h})(mB_{ED}^{\alpha h}(1-m)\alpha_h^2 + (1-m)B_{ED}^{\alpha h}(1-m)\alpha_l^2) - \hat{A}
\]

\[
= (1-B_{ED}^{\alpha h})B_{ED}^{\alpha h}(1-m)(m\alpha_h^2 + (1-m)\alpha_l^2)
\]

\[
- \frac{r}{2}(1-m)\sigma_l^2(B_{ED}^{\alpha h})^2 + \frac{1}{2}(B_{ED}^{\alpha h})^2(1-m)^2\alpha_l^2 + m(1-m)\alpha_h^2(B_{ED}^{\alpha h})^2
\]

\[
= B_{ED}^{\alpha h}(1-m)(m\alpha_h^2 + (1-m)\alpha_l^2)
\]

\[
- (B_{ED}^{\alpha h})^2((1-m)(m\alpha_h^2 + (1-m)\alpha_l^2) + \frac{r}{2}(1-m)\sigma_l^2 - \frac{1}{2}(1-m)^2\alpha_l - m(1-m)\alpha_h^2)
\]

\[
= B_{ED}^{\alpha h}(1-m)(m\alpha_h^2 + (1-m)\alpha_l^2) - (B_{ED}^{\alpha h})^2\frac{1}{2}(1-m)(r\sigma_l^2 + (1-m)\alpha_l^2)
\]

For the low-type principal not to deviate, it must be that:

\[
\frac{\epsilon_l^4(1-m)}{2(r\sigma_l^2 + (1-m)\alpha_l^2)} - V_l \geq 0
\]

This implies that \(B_{ED}^{\alpha h} \notin (B_l, B_r)\), where:

\[
B_l = \frac{m\alpha_h^2 + (1-m)\alpha_l^2 + \sqrt{\Delta}}{r\sigma_l^2 + (1-m)\alpha_l^2}
\]

\[
B_r = \frac{m\alpha_h^2 + (1-m)\alpha_l^2 - \sqrt{\Delta}}{r\sigma_l^2 + (1-m)\alpha_l^2}
\]

\[
\Delta = (m\alpha_h^2 + (1-m)\alpha_l^2)^2 - 4\frac{1}{2}(r\sigma_l^2 + (1-m)\alpha_l^2)\frac{\epsilon_l^4}{2(r\sigma_l^2 + (1-m)\alpha_l^2)}
\]

\[
= (m\alpha_h^2 + (1-m)\alpha_l^2)^2 - \frac{4\epsilon_l^4}{2(r\sigma_l^2 + (1-m)\alpha_l^2)}
\]

\[
= m(\alpha_h^2 - \alpha_l^2)(m\alpha_h^2 - (m-2)\alpha_l^2)
\]

It follows that \(\sqrt{\Delta} < m(\alpha_h^2 - \alpha_l^2)\). Therefore:

\[
\sqrt{\Delta} < m(\alpha_h^2 - \alpha_l^2) \frac{m\alpha_h^2 + (1-m)\alpha_l^2 + m(\alpha_h^2 - \alpha_l^2)}{r\sigma_l^2 + (1-m)\alpha_l^2} = \frac{\alpha_h^2 + 2\alpha_h m - 2\alpha_l m}{r\sigma_l^2 + (1-m)\alpha_l^2} > B_{FI}^{\alpha h}
\]

\[
\frac{\epsilon_l^4}{2(r\sigma_l^2 + (1-m)\alpha_l^2)} > \frac{\alpha_l^2}{r\sigma_l^2 + (1-m)\alpha_l^2} = B_{FI}^{\alpha h} < B_{E}^{\alpha h}
\]

Therefore the full-information contract \(B_{E}^{\alpha h}\) is not incentive-compatible. In addition, let us compare whether \(B_l\) or \(B_r\) is the contract that is most preferred by the high type. Because the objective function of the high type is a parabola,
it is sufficient to compare \( B_{F_1}^{\alpha_h} \) to \((B_{I} + \overline{B}_{I})/2\).

\[
\frac{B_{F_1} + \overline{B}_{F_1}}{2} - B_{F_1}^{\alpha_h} = \frac{m\alpha_h^2 + (1-m)\alpha_l^2}{r\sigma_l^2 + (1-m)\alpha_l^2} - \frac{\alpha_h^2}{r\sigma_l^2 + (1-m)\alpha_l^2} \\
= \frac{(m\alpha_h^2 + (1-m)\alpha_l^2)(r\sigma_l^2 + (1-m)\alpha_h) - \alpha_h^2(r\sigma_l^2 + (1-m)\alpha_l)}{(r\sigma_l^2 + (1-m)\alpha_l^2)(r\sigma_l^2 + (1-m)\alpha_h)} \\
= \frac{(\alpha_h^2 - \alpha_l^2)(1-m)(m\alpha_h^2 - r\sigma_l^2)}{(r\sigma_l^2 + (1-m)\alpha_l^2)(r\sigma_l^2 + (1-m)\alpha_h)}
\]

When this term is positive, i.e. \( m \geq r\sigma_l^2/\alpha_h^2 \), the closest \( B \) to \( B_{F_1}^{\alpha_h} \) is \( B_{F_1} \) and therefore it is also the most preferred by the high type; otherwise, the most preferred contract is \( \overline{B}_{F_1} \).

2. We consider next whether the high type would deviate to offer the contract preferred by the low type. Let \( \mathcal{V}_h \) denote the expected surplus received by the high type after this deviation. By Lemma 3.1, the fixed compensation offered by the high type must be:

\[
\hat{A}_h = \frac{r}{2} (1-m)\sigma_l^2(B_{F_1}^{\alpha_l})^2 - \frac{1}{2} (B_{F_1}^{\alpha_l})^2 (1-m)\alpha_l^2 - m(1-m)\alpha_l^2 (B_{F_1}^{\alpha_l})^2 \\
= \frac{4}{(r\sigma_l^2 + (1-m)\alpha_l^2)^2} \frac{1-m}{2} \left(-2m\alpha_l^2 - 2(1-m)\alpha_h^2 - r\sigma_l^2 + (1-m)\alpha_l^2 + 2m\alpha_l^2\right) \\
+ \frac{\alpha_l^2}{r\sigma_l^2 + (1-m)\alpha_l^2} \frac{1-m}{2} \left(2m\alpha_l^2 + 2(1-m)\alpha_h^2\right) \\
= \frac{(1-m)\alpha_l^2}{2(r\sigma_l^2 + (1-m)\alpha_l^2)^2} \left(-\alpha_l^2 r\sigma_l^2 - \alpha_l^2 (1-m)\alpha_h^2 + 2(\sigma_l^2 + (1-m)\alpha_l^2)(m\alpha_l^2 + (1-m)\alpha_h^2)\right)
\]

\[
\mathcal{V}_{ED} - \mathcal{V}_h = \frac{1-m}{2} (B_{F_1}^{\alpha_h})^2 ((1-m)\alpha_h^2 + r\sigma_l^2) - \frac{1-m}{2} 2B_{F_1}^{\alpha_h} \alpha_h^2 - \mathcal{V}_h
\]

To begin with, note that the incentive coefficient most likely to satisfy this constraint is the one that maximizes the surplus of the high-type principal. Therefore, when considering incentive-compatibility of the low type, one can restrict the attention to \( B_I \) if \( m \geq r\sigma_l^2/\alpha_h^2 \), or \( B_I \) otherwise.

We discuss next the existence of a fully-separating equilibrium. First, assume first that \( m \geq r\sigma_l^2/\alpha_h^2 \), then substituting \( B_I \) in the incentive-compatibility condition, one obtains that:

\[
\mathcal{V}_{ED}^{\alpha_h} - \mathcal{V}_h|_{B=B_I} = \frac{(\alpha_h^2 - \alpha_l^2)(1-m)^3 \left(m^2 \alpha_h^2 + m \left((1-m)\alpha_l^2 - r\sigma_l^2 - \sqrt{\Delta}\right)\alpha_l^2 - \alpha_l^4 m + \alpha_l^2 m r\sigma_l^2 + \sqrt{\Delta} r\sigma_l^2\right)}{\alpha_l^2 (1-m) - r\sigma_l^2)^2}
\]

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This Equation has two potential roots $\alpha_1$ and $\alpha_2$ in $(0, \alpha_h)$ which are given by:\footnote{We also verified that $\alpha_2$ is always real and in $(0, \alpha_h)$ when $m \geq r\sigma^2_2/\alpha_h^2$. On the other hand, when $m \geq r\sigma^2_2/\alpha_h^2$, there may be parameter values for which a fully-separating equilibrium always exists (i.e., the roots $\alpha_1$ and $\alpha_2$ are not real). We use here the notation $\notin$ in its strict sense, given that $\alpha_1$ may not be in a segment for which the endpoints are imaginary.}

$$
\begin{align*}
\alpha_1 &= \sqrt{-\left(m - \frac{1}{2}\right)\alpha_h^2 + r\sigma^2 - \sqrt{(2m\sigma^2_2 - \alpha_h^2(2m - 1))^2 + 4m\sigma^2_2(\alpha_h^2m - r\sigma^2_2)}} \\
\alpha_2 &= \sqrt{-\left(m - \frac{1}{2}\right)\alpha_h^2 + r\sigma^2 + \sqrt{(2m\sigma^2_2 - \alpha_h^2(2m - 1))^2 + 4m\sigma^2_2(\alpha_h^2m - r\sigma^2_2)}}
\end{align*}
$$

When $m > r\sigma^2_2/\alpha_h^2$, the root $\alpha_1$ is imaginary while the second root $\alpha_2$ is always in $(0, \alpha_h)$. It follows that incentive-compatibility is satisfied for $\alpha_1 \geq \alpha_2$.

Similarly, when $m < r\sigma^2_2/\alpha_h^2$,

$$
\frac{V_{\alpha_h}^{a_{\alpha}} - V_h|_{B=\pi_l}}{(\alpha_h^2 - \alpha_1^2)(1 - m)^2 \left(m^2\alpha_h^4 + m((1 - m)\alpha_h^2 - r\sigma^2 + \sqrt{\Delta})\alpha_h^2 - \alpha_h^4m + m^2r\sigma^2 - \sqrt{\Delta}r\sigma^2\right)} = (\alpha_h^2(1 - m) - r\sigma^2)^2
$$

This inequality is positive if and only if $\alpha_1 \notin (\alpha_1, \alpha_2)$, where $alpha_1$ and $\alpha_2$ are the roots defined previously.$\square$

**Proof of Proposition 4.3:** To prove the result, it is sufficient to verify that: 1. conditional on choosing the Full Information contract corresponding type $\alpha_h$, the agent would not participate; 2. the principal with type $\alpha_h$ would not offer the Full Information contract corresponding to type $\alpha_1$.

1. Let $\tilde{A}$ denote the minimum fixed compensation required by the agent to participate to the $\alpha_h$ Full-Information contract if the actual productivity is $\alpha_1$.

$$
\begin{align*}
\tilde{A} &= \frac{r}{2}(1 - m)\sigma_2^2(B_{FI}^\alpha)^3 - \frac{1}{2}(B_{FI}^\alpha)^2(1 - m)^2\alpha_1^2 - m(1 - m)\alpha_h^2(B_{FI}^\alpha)^2 \\
&= A_{FI}^{\alpha_h} + \frac{1}{2}(B_{FI}^\alpha)^2(1 - m)^2(\alpha_h^2 - \alpha_1^2) > A_{FI}^{\alpha_h}
\end{align*}
$$

Therefore, the agent would not participate to the contract.

2. Assume that the high-type principal offers the contract $(A_{FI}^{\alpha_1}, B_{FI}^{\alpha_1})$ and let $\hat{V}$ be the expected surplus after this deviation occurs.

$$
\hat{V} = (1 - B_{FI}^{\alpha_1})(m\alpha_lB_{FI}^{\alpha_1}(1 - m)\alpha_1 + (1 - m)\alpha_hB_{FI}^{\alpha_1}(1 - m)\alpha_h) - A_{FI}^{\alpha_1}
$$
Then:

\[
\frac{\partial \tilde{V}}{\partial \alpha_h} = 2B_{F_I}^2(1 - B_{F_I}^2)\alpha_h(1 - m)^2 \\
= 2\frac{\alpha_h^2}{r\sigma^2 + (1 - m)\alpha_h^2} \frac{r\sigma^2 - m\alpha_h^2}{r\sigma^2 + (1 - m)\alpha_h^2}\alpha_h(1 - m)^2 \\
= 2\frac{\alpha_h^2(r\sigma^2 - m\alpha_h)}{(r\sigma^2 + (1 - m)\alpha_h^2)^2}\alpha_h(1 - m)^2
\]

Recall from Proposition 3.1 that:

\[
V_{F_I}^{\alpha_h} = \frac{4\alpha_h^4(1 - m)}{2(r\sigma^2 + (1 - m)\alpha_h^2)}
\]

Therefore:

\[
\frac{\partial V_{F_I}^{\alpha_h}}{\partial \alpha_h} = 4\alpha_h^4(1 - m)(r\sigma^2 + (1 - m)\alpha_h^2) - 2\alpha_h(1 - m)^4\alpha_h^4(1 - m) \\
= 2\alpha_h(1 - m)^2\alpha_h^{4/2} + \alpha_h^2r\sigma^2/(1 - m) \\
> 2\alpha_h(1 - m)^2\frac{\alpha_h^{4/2} + \alpha_h^2r\sigma^2/(1 - m)}{(r\sigma^2 + (1 - m)\alpha_h^2)^2} \\
> 2\alpha_h(1 - m)^2\frac{\alpha_h^2(r\sigma^2 - m\alpha_h)}{(r\sigma^2 + (1 - m)\alpha_h^2)^2} = \frac{\partial \tilde{V}}{\partial \alpha_h}
\]

The function \( \tilde{V} - V_{F_I}^{\alpha_h} \) is equal to zero at \( \alpha_h = \alpha_1 \), and otherwise is decreasing in \( \alpha_h \), therefore \( \tilde{V} > V_{F_I}^{\alpha_h} \). It follows that the high-type principal would not offer the low-type contract. □

**Bibliography**


**Baldenius, Tim and Xiaojing Meng**, “Signaling Firm Value to Active Investors,” 2008.


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