Ironing out the wrinkles in executive compensation:  
Linking incentive pay to average stock prices*

Yisong S. Tian  
Schulich School of Business  
York University  
4700 Keele Street  
Toronto, Ontario  
Canada, M3J 1P3  
Email: ytian@schulich.yorku.ca  
Phone: 416-736-2100 ext.77943

November 12, 2011

* We thank Phelim Boyle, Don Chance, Mark Flannery, Cam Harvey, Mark Huson, Marti Subrahmanyan, Jun Yang, the participants of the 2010 annual meeting of the Financial Management Association, the 2010 Laurier Annual Finance Conference, and the 2011 annual meeting of the Northern Finance Association, and the seminar participants at the Louisiana State University for helpful comments and suggestions. The financial support of the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.
Ironing out the wrinkles in executive compensation: 
Linking incentive pay to average stock prices

Abstract

Traditional stock option grant is the most common form of incentive pay in executive compensation. Applying a principal-agent analysis, we find this common practice suboptimal and firms are better off linking incentive pay to average stock prices. Holding the cost of the option grant to the firm constant, Asian stock options are more cost effective than traditional stock options and provide stronger incentives to increase stock price. More importantly, the improvement is achieved with little impact on the option grant’s risk incentives (after adjusting for option cost). Finally, averaging also improves the value and incentive effects of indexed stock options.

JEL classifications: G13; G30; J33; M52

Key words: Executive compensation; Optimal contracting; Executive stock options; Cost effectiveness; Incentive effects; Asian stock options; Indexed stock options.

1. Introduction

U.S. corporations typically link their top executives’ pay to long-term stock performance by granting them stock options, restricted stock or a combination of both. In the case of stock options, the overwhelming majority are in the form of traditional stock options which are typically granted at the money and have a ten-year term. Despite their popularity, both forms of incentive pay have been criticized for not providing adequate alignment of interests between executives and shareholders (e.g., Meulbroek, 2001; Hall and Murphy, 2003). Traditional stock options are not cost effective as executives generally value them at a large discount from their market value (due to the executive’s risk aversion and hedging restrictions). They may also misalign risk incentives due to the convex payoff structure. Options derive their payoffs exclusively from the right side of the stock return distribution while stock prices are influenced by events that affect both sides. This difference in payoff structure may lead to different attitudes toward risk between executives and shareholders. In comparison, restricted stock is more cost effective and provides better incentive alignment than traditional stock options do. However,
restricted stock provides weaker incentives to increase stock price than traditional stock options do. Holding the level of required incentives constant, it may be more costly for the firm to grant restricted stock than traditional stock options. Neither seems to be ideal instrument for incentive pay.

In this paper, we argue that the common practice of linking incentive pay to stock prices is suboptimal and firms are better off linking incentive pay to average stock prices. Applying a standard principal-agent model, we analyze the optimal level of equity incentives if they are based on either the stock price or the average stock price. We show that equity incentives are more cost effective if they are based on the average stock price. Holding the incentive strength constant, the cost of providing the equity incentive is lower if incentive pay is linked to the average stock price instead of the stock price. For a typical U.S. firm,¹ the cost savings are more than 50% if average stock price-based equity incentives are adopted. Stock price-based incentive scheme is thus dominated by average stock price-based incentive scheme.²

This optimal contracting result has important implications for executive compensation. Instead of granting traditional stock options, firms should consider granting Asian stock options.³ Asian stock options (with a fixed strike price) are not only more cost effective but also more incentive effective than traditional stock options. By more cost effective, we mean that Asian stock options are discounted less from their market values by risk-averse executives than traditional stock options are. In other words, risk-averse executives have a higher subjective value for Asian stock options than for traditional stock options if the total market value of the option grant is held constant. As the cost of the option grant to the firm is identical or proportional to its market value, Asian stock options are thus more cost effective than traditional stock options are.

¹ We define the typical firm as one with a 4% excess return (over the risk-free rate), stock return volatility of 30% and an executive with relative risk aversion of 3.

² Note that we do not argue that average stock price-based incentive pay is the best among all possible incentive schemes. We simply argue that it dominates the commonly adopted stock price-based incentive scheme.

³ Although the idea of using Asian stock options in executive compensation has been advocated by practitioners (e.g., Chhabra, 2008), there has been no rigorous analysis of this alternative form of incentive pay. Our paper fills this gap in the literature and evaluates the value and incentive effects of Asian stock option grants.
Of course, being more cost effective is not enough. After all, cash pay (e.g., salary and bonus) is the most cost effective form of compensation but provides no incentives once it is paid out. Stock options, though less cost effective than cash pay, do provide incentives for executives to increase stock price and align their interests with those of shareholders. The key question is thus whether or not Asian stock options provide stronger incentives to increase stock price than traditional stock options do. The answer is emphatically yes. Holding the cost of the option grant constant, Asian stock options provide much stronger incentives to increase stock price than traditional stock options do. Under conditions that mimic the typical corporate contracting environment, we show that Asian stock options generally provide incentives that are roughly 69%–87% stronger than those provided by traditional stock options. As a result, it costs the firm much less to provide a given level of equity incentives if Asian stock options instead of traditional stock options are granted. Such improvement in incentive strength should be attractive to most firms.

Why is linking incentive pay to average stock price more cost effective and incentive effective? One important reason is that averaging reduces volatility substantially which is beneficial to risk-averse executives. The reduced volatility of incentive pay increases its subjective value to the executive, making this form of incentive pay more cost effective. Although averaging also reduces the expected payoff, it is a tradeoff that most risk-averse executives prefer given the disproportionally large reduction in volatility. In addition, linking incentive pay to average stock prices also improves the alignment of interests between executives and shareholders.\(^4\) When compensation is linked directly to the stock price, the executive is motivated to maximize stock performance over a specific horizon. The particular path the stock price takes may be less important to the executive as long as the stock price reaches the desired level in the end. In comparison, shareholders prefer steady growth to volatile swings if the overall stock return is the same. Such differences in preference are even more magnified if incentive pay is structured in the form of stock options instead of restricted stock given the asymmetry of option payoffs. Linking incentive pay to the average stock price helps

\(^4\) To improve alignment of interests between the executive and shareholders, incentive pay should be linked to the geometric average stock price instead of the arithmetic average stock price. This is because geometric averaging penalizes mean preserving spreads while arithmetic averaging does not. This interesting feature makes geometric averaging a better choice for setting up option payoffs. We thus adopt geometric averaging throughout this paper.
bridge this gap in preference between executives and shareholders. This is because the (geometric) average stock price is inversely related to stock return volatility if the expected return is held constant. Intuitively, a mean preserving spread (i.e., higher volatility with the same mean) leads to lower (geometric) average stock price since an increase in stock price followed by a decrease in stock price of the same amount ($\Delta S$) leads to a decline in average stock price (i.e., $\sqrt{(S + \Delta S)(S - \Delta S)} < S$). Linking incentive pay to average stock prices thus improves interest alignment between the executives and shareholders, with both groups preferring steady growth over volatile ones (holding the expected return constant). Restricted stock provides a similar type of interest alignment but has weaker incentive effects than stock options. Asian stock options thus improve both incentive alignment and incentive strength and provide a better way to compensate executives than both traditional stock options and restricted stock.

Another potential benefit of linking incentive pay to the average stock price is that it makes it more difficult for executives to profit from opportunistic manipulation of stock prices. Through fraudulent accounting practices and other dubious actions, executives can manipulate stock prices in order to gain extra payoffs from their option and restrict stock holdings (e.g., Johnson et al., 2009). Although Asian stock options are unlikely to prevent executives from committing fraud, they should discourage it since the payoffs of Asian stock options are based on the average stock price over the option’s life and thus more difficult or costly to manipulate than the payoffs of traditional stock options.

A surprising finding in our analysis is that the gains in cost effectiveness and incentive strength by Asian stock options are achieved without any material change to the option grant’s risk incentives. This appears counter intuitive at first since averaging reduces stock return volatility. Because Asian stock options are less valuable than the corresponding traditional stock options, firms must grant more than one Asian stock options to replace one traditional stock option. With more Asian stock options granted than traditional stock options, the overall impact on risk incentives is no longer obvious. Holding the cost of the option grant to the firm constant, we show that both traditional and Asian stock options provide very similar risk incentives to risk-averse executives. A change in the firm’s risk level leads to similar changes in the value of the executive’s total wealth regardless of the type of options granted (Asian or traditional). This is true for incentives to change systematic risk (while holding idiosyncratic risk constant), incentives to change idiosyncratic risk (while holding systematic risk constant), and incentives to
change both systematic and idiosyncratic risks simultaneously. As there is no consensus in the literature on whether stock options create too much or too little risk incentives (e.g., Carpenter, 2000; Ross, 2004), the use of Asian stock options is not likely to change that due to the similarity in risk incentives they provide in comparison with traditional stock options. Such similarity in risk incentives also ensures that Asian stock options dominate traditional stock options as incentive pay because they are more cost effective and provide stronger incentives to increase stock price.

Interestingly, averaging also substantially improves the value and incentive effects of indexed stock options. By indexing the strike price to a benchmark stock market index, indexed options filter out market performance and pay off for the executive only if the firm outperforms the benchmark index. However, indexed stock options have been criticized for being less cost effective than traditional stock options (e.g., Hall and Murphy, 2003) and there is a complete lack of interest from corporate compensation committees to adopt such options. This may change if average prices are incorporated in the payoff structure of indexed options. In particular, we show that indexed Asian stock options (based on average stock price and average index level) are much more cost effective than indexed traditional options (those introduced by Johnson and Tian, 2000a) and provide much stronger incentives to increase stock price. In fact, indexed Asian stock options even dominate traditional stock options in both cost effectiveness and incentive strength (holding the cost of the option grant fixed). As a result, we can no longer dismiss indexing on the basis of cost effectiveness. It is instead a matter of choice between filtering and not filtering out market performance in the design of incentive pay.

In addition, the choice of strike price remains an important factor in the determination of incentive levels provided by stock options. As is well known in prior research (e.g., Hall and Murphy, 2000; Palmon et al., 2005), the maximum level of incentives to increase stock price is typically achieved by granting stock options in the money. Holding the cost of the option grant fixed, the common practice of granting options at the money is suboptimal. This is not surprising since in-the-money options have a greater chance of expiring in the money, making them more attractive to risk-averse executives. Even though this conclusion is originally based on the analysis of traditional stock options, we show that it is also true for Asian stock options, indexed traditional stock options, and indexed Asian stock options. Granting the option at the money is thus suboptimal for all four types of options and the incentive levels can be substantially less
than those provided by in-the-money options. The exact optimal strike price level depends on, among other things, the type of options granted. For traditional and indexed traditional stock options, the optimal strike price is usually smaller than it is for the corresponding Asian and indexed Asian stock options, respectively. In other words, firms should choose a lower strike price for traditional (indexed traditional) options than for Asian (indexed Asian) options in order to maximize their incentive effects.

The rest of the paper proceeds as follows. Section 2 applies a standard principal-agent model to analyze the characteristics of the optimal contract when equity incentives are linked to average stock prices in comparison with those when equity incentives are linked directly to stock prices. Section 3 examines Asian and indexed Asian stock options and compares their valuation with that of the corresponding traditional and indexed traditional stock options. In Section 4, we evaluate and compare the incentive effects provided by the four types of stock options. Robustness and other concerns are discussed in Section 5. The final section concludes.

2. Linking incentive pay to average stock prices

We first characterize the optimal contract when the firm compensates its executive with a fixed salary and equity incentives in a standard principal-agent model. We do not ask the more fundamental question whether the optimal contract should or should not include stock or stock options (e.g., Dittmann and Maug, 2007). Instead, the focus of the analysis is on the comparison of equity incentives linked to the average stock price vs. equity incentives linked to the stock price and their relative effectiveness in motivating the executive. For this reason, we adopt a relatively simple principal-agent model (e.g., Edmans et al., 2009) so that it can be used to analyze equity incentives linked to the average stock price as well as those linked to the stock price. This is unlikely to affect the relative advantages or disadvantages between the two forms of equity incentives in our analysis.

The firm compensates the executive with shares of either its stock or a tracking stock that mimics the performance of the (geometric) average stock price. The executive is both risk averse

---

5 Although more sophisticated principal-agent models such as Dittmann and Maug (2007) and Armstrong et al. (2010) are clearly superior theoretically, they are also more difficult to apply in our setting when equity incentives are linked to average stock prices. There is also no guarantee that a more sophisticated model is better at explaining the realities of executive compensation. For example, while stock options are prevalent in CEO contracts in practice, they are not even part of the optimal contract in Dittmann and Maug (2007).
and effort averse. Given the equity incentives provided by the firm, the executive chooses the level of his or her effort in order to maximize the expected utility of his or her own terminal wealth. The firm’s stock price is assumed to follow a geometric Brownian motion:

$$\frac{dS_t}{S_t} = (\mu - q)dt + \sigma dz_t$$

(1)

where \( \mu \) is the expected rate of return, \( q \) is the dividend yield, \( \sigma \) is volatility and \( z_t \) is a standard Brownian motion. Define the (geometric) average stock price over the period \([0, t]\) as:

$$\hat{S}_t = \exp\left(\frac{1}{t} \int_0^t \log S_u du\right)$$

It is well known from prior research (e.g., Kemna and Vorst, 1990) that the average stock price also follows a geometric Brownian motion but with different drift and volatility terms:

$$\frac{d\hat{S}_t}{\hat{S}_t} = (\hat{\mu} - \hat{q})dt + \hat{\sigma}\hat{d}z_t$$

(2)

where

$$\hat{\mu} = \frac{1}{2}(\mu + r),$$

$$\hat{q} = \frac{1}{2}(r + q + \frac{1}{6}\sigma^2),$$

$$\hat{\sigma} = \frac{\sigma}{\sqrt{3}}.$$

\( r \) is the risk-free rate, and \( \hat{z}_t \) is also a standard Brownian motion.

Note that \( \hat{S}_0 = S_0 \) by definition, so the average stock price also has an initial value of \( S_0 \).

If a tracking stock were issued to mimic the performance of the average stock price, its price would be identical to the firm’s stock price initially but has lower volatility and grows at a lower rate

$$\hat{\mu} - \hat{q} = \frac{1}{2}(\mu - q - \frac{1}{6}\sigma^2) < \mu - q$$

than the firm’s stock price does. More importantly, any increase in stock volatility would reduce the growth rate (or expected return) of the tracking stock (which exactly mimics the performance of the average stock price). This means that executives have lower incentives to increase risk if
their compensation is linked to the average stock price as opposed to the stock price itself (assuming they are provided with the same number of shares of either the stock or the tracking stock).

Another important feature of the average stock price is its lower volatility. Its volatility \( \left( \sigma / \sqrt{3} \right) \) is roughly 42\% lower than the stock’s volatility \( (\sigma) \). Such a drastic reduction in volatility suggests that risk-averse executives may prefer to have their incentive pay tied to the average stock price instead of stock price itself, holding everything else constant. Of course, the growth rate (or expected return) of the average stock price is also substantially lower than the corresponding growth rate of the stock price. There is thus a tradeoff between lower volatility and lower return.

Is the large reduction in volatility sufficient to compensate for the lower growth rate? If it is, a tracking stock mimicking the average stock price may be more cost effective for compensating executives than the stock itself. To verify if this is indeed the case, we analyze the optimal contract between the firm and its executive with the equity incentives linked to either the stock price or the average stock price. A comparison of the two optimal contracts should provide answers to our question. For simplicity, we ignore the salary component of the executive’s compensation and focus only on the equity incentives (e.g., Ross, 2004). This is not unreasonable as cash pay is a small fraction of a typical CEO’s annual compensation.

The executive is risk averse with constant relative risk aversion:

\[ u(w) = \frac{w^{1-\gamma}}{1-\gamma} \]  

(3)

where \( w \) is the executive’s terminal wealth on a target date \( T \) and \( \gamma \geq 0 \) is the coefficient of relative risk aversion. Positive \( \gamma \) indicates risk aversion while risk neutrality is specified by \( \gamma = 0 \). The executive is also effort averse which reduces his utility via a multiplicative cost of effort function (e.g., Edmans et al., 2009):

\[ u[w \cdot g(a)] \]

where \( a \) is a proxy for the executive’s effort, normalized to have its value between 0 and \( \bar{a} \) \( (0 \leq a \leq \bar{a} < 1) \), and the cost of effort function is specified as:

\[ g(a) = \exp[\delta(\bar{a} - a)^\alpha] \]  

with \( \alpha \geq 1 \). Our analysis and results of the optimal contract are not materially affected.

---

6 We also consider a more general form of the cost of effort function \( g(a) = \exp[\delta(\bar{a} - a)^\alpha] \) with \( \alpha \geq 1 \). Our analysis and results of the optimal contract are not materially affected.
\[ g(a) = \exp[\delta(\bar{a} - a)], \quad \delta > 0 \] (4)

In order to motivate the executive to exert the most effort (i.e., choosing \( a = \bar{a} \)), the firm provides \( \lambda \) shares of either the firm’s stock or the tracking stock that mimics the performance of the average stock price. Both types of shares are restricted in the sense that the executive cannot sell them or cash out until a future date \( T \). The executive’s effort positively impacts the firm’s expected stock returns, which is summarized by the following productivity function:

\[ \mu(a) = (1 + a - \bar{a})\mu \] (5)

A stronger effort (i.e., a greater \( a \)) leads to a higher expected return (\( \mu(a) \)) which is beneficial to both the executive and shareholders. The executive has no outside wealth and maximizes the expected utility of his or her terminal wealth at the end of \( T \) years.

If the executive is provided with \( \lambda \) shares of stock as compensation, his or her terminal wealth is given by:

\[ W_T = \lambda S_0 \exp\left\{ (1 + a - \bar{a})\mu - \frac{1}{2}\sigma^2 \left[ T + \sqrt{T}\varepsilon_T \right] \right\}, \]

where \( \varepsilon_T \) is a standard normal variate. Given the executive’s choice of effort level \( (a) \), his or her expected utility is thus:

\[ EU = E\{u[W_T g(a)]\}. \]

Solving the expectations analytically (i.e., integrating out the expectations operator), we have:

\[ EU = u\left\{ \lambda S_0 \exp\left[ \mu - \frac{1}{2}\gamma\sigma^2 \left( T + (a - \bar{a})(\mu T - \delta) \right) \right] \right\}. \]

The executive is thus motivated to provide the best effort (i.e., \( a = \bar{a} \)) as long as he or she is not too effort averse (i.e., \( \delta < \mu T \) which we assume is always true) and a positive number of shares is provided as compensation. Given the choice of effort (i.e., \( a = \bar{a} \)), the expected utility is:

\[ EU = u\left\{ \lambda S_0 \exp\left[ \mu - \frac{1}{2}\gamma\sigma^2 T \right] \right\}. \]

The optimal contract thus calls for the firm to provide just enough shares to meet the executive’s reservation utility \( (U_0) \):

\[ U_0 = u\left\{ \lambda S_0 \exp\left[ \mu - \frac{1}{2}\gamma\sigma^2 T \right] \right\}. \]
This participation constraint can be stated more conveniently using certainty equivalent value:

\[
TCE_0 \exp(rT) = \bar{\lambda}_i S_0 \exp\left[\left(\mu - \frac{1}{2} \gamma \sigma^2\right)T\right],
\]

where \(TCE_0\) is the cash pay today that would provide the executive the exact reservation utility required for him or her to take the job. The total number of shares called for by the optimal contract is thus:

\[
\bar{\lambda}_i = \frac{TCE_0 \exp(rT)}{S_0} \exp\left[-\left(\mu - \frac{1}{2} \gamma \sigma^2\right)T\right].
\]

What if the executive is compensated with \(\lambda\) shares of the tracking stock that mimics the performance of the average stock price instead? The terminal payoff of the equity portfolio at time \(T\) is now worth:

\[
W_T = \lambda S_0 \exp\left[\left(\hat{\mu}(a) - \hat{q} - \frac{1}{2} \hat{\sigma}^2\right)T + \hat{\sigma} \sqrt{T} \epsilon_T\right],
\]

where

\[
\hat{\mu}(a) = \frac{1}{2} [(1 + a - \bar{a}) \mu + r]
\]

is the expected return of the average stock price given the executive’s choice of effort \(a\). By substitution, we can simplify the terminal payoff as:

\[
W_T = \lambda S_0 \exp\left\{\frac{1}{2} \left[(1 + a - \bar{a}) \mu - q - \frac{1}{2} \sigma^2\right]T + \sigma \sqrt{T/3} \epsilon_T\right\}.
\]

The expected utility of the terminal wealth is thus:

\[
EU = u\left\{\lambda S_0 \exp\left[\frac{1}{2} \left(\mu - q - \frac{1}{6} (1 + 2\gamma) \sigma^2\right)T + (a - \bar{a}) \left(\frac{1}{2} \mu T - \delta\right)\right]\right\}.
\]

Similar to stock price-based equity incentives, the executive is motivated to give the best effort (i.e., \(a = \bar{a}\)) as long as he or she is not too effort averse (i.e., \(\delta < 0.5\mu T\) which we assume is always true) and a positive number of shares is provided as compensation. Given the choice of effort (i.e., \(a = \bar{a}\), the expected utility is:

\[
EU = u\left\{\lambda S_0 \exp\left[\frac{1}{2} \left(\mu - q - \frac{1}{6} (1 + 2\gamma) \sigma^2\right)T\right]\right\}.
\]
Again, the optimal contract calls for the firm to provide just enough number of shares to meet the executive’s reservation utility:

$$U_0 = u\left(\overline{\mathcal{X}}_2S_0 \exp\left[\frac{1}{2}\left(\mu - q - \frac{1}{6}(1 + 2\gamma)\sigma^2\right)T\right]\right),$$

or equivalently:

$$TCE_0 \exp(rT) = \overline{\mathcal{X}}_2S_0 \exp\left[\frac{1}{2}\left(\mu - q - \frac{1}{6}(1 + 2\gamma)\sigma^2\right)T\right].$$

The total number of shares called for by the optimal contract is thus:

$$\overline{\mathcal{X}}_2 = \frac{TCE_0 \exp(rT)}{S_0} \exp\left[-\frac{1}{2}\left(\mu - q - \frac{1}{6}(1 + 2\gamma)\sigma^2\right)T\right].$$

While the optimal contract has many similarities under both compensation schemes (i.e., stock vs. average stock), it also has some important differences. Under both schemes, the optimal level of equity incentives is negatively related to the stock’s expected return ($\mu$), positively related to the stock’s volatility ($\sigma$), and positively related to the executive’s degree of risk aversion ($\gamma$). More importantly, the executive is motivated to provide the best effort under either compensation scheme. Despite these similarities, there is a key difference between the two schemes which is their cost to the firm. The scheme with a lower cost is the more effective one as the incentive effects of the two schemes are identical.

If equity incentives are linked to the stock price, the total cost of the optimal contract to the firm is simply the required number of shares multiplied by the stock price:

$$TC_1 = \overline{\mathcal{X}}_2S_0 = [TCE_0 \exp(rT)] \exp\left[-\left(\mu - \frac{1}{2}\gamma\sigma^2\right)T\right].$$

If equity incentives are linked to the average stock price, the total cost of the optimal contract to the firm is determined similarly except for the loss of future dividends:

$$TC_2 = \overline{\mathcal{X}}_2S_0 \exp(-qT) = [TCE_0 \exp(rT)] \exp\left[-\frac{1}{2}\left(\mu + r - \frac{1}{3}\gamma\sigma^2\right)T\right].$$

The cost ratio of the two compensation schemes is thus:

$$\frac{TC_1}{TC_2} = \exp\left[-\frac{1}{2}\left(\mu - r - \frac{2}{3}\gamma\sigma^2\right)T\right] \quad (6)$$
The average stock price scheme is more effective as long as the executive is sufficiently risk averse:

\[ \gamma > \gamma^* = 1.5 \times \frac{\mu - r}{\sigma^2}, \]

where \( \gamma^* \) is a threshold level of risk aversion where the cost to the firm is the same for both types of equity incentives.

The relevant question is then how high is the level of this critical threshold of risk aversion level (\( \gamma^* \)). For a typical firm with moderate stock return characteristics, say \( \mu = 0.08, r = 0.04 \) and \( \sigma = 0.3 \), the critical threshold level of risk aversion (\( \gamma^* \)) is only 0.67. This means that the firm would be better off by linking equity incentives to the average stock price as opposed to the stock price if its executive has a coefficient of relative risk aversion greater than 0.67. As \( \gamma = 0 \) indicates risk neutrality, \( \gamma = 0.67 \) characterizes someone with a very low degree of risk aversion.

For a more typical risk-averse executive with a relative risk aversion of 3, the cost ratio (\( TC_1/TC_2 \)) of the equity incentives is 2.01. This means that the cost of providing stock price-based equity incentives can be more than twice as large as the cost of providing average stock price-based equity incentives.

In Figure 1, we provide a more extensive illustration of the critical threshold level of risk aversion. In particular, we show that the risk-averse executive’s preference for the average stock price is much more general than the typical case presented previously. In Panel A, we plot the critical threshold of risk aversion level against the stock’s excess return (\( \mu - r \)) while keeping the stock’s volatility fixed at 30%. In Panel B, we plot the critical threshold against the stock’s volatility (\( \sigma \)) while holding the stock’s excess return fixed at 4%. In both cases, it is clear that the critical threshold is generally quite low, consistently less than 2.0, over a wide range of plausible parameter values. We thus conclude that the average stock price provides a better risk-return tradeoff than the stock price itself and most executives are likely to prefer the average stock price over the stock price. It makes sense for firms to link incentive pay to average stock prices as opposed to stock prices.

3. The use of Asian stock options in executive compensation
The optimal contracting analysis in the previous section suggests that linking incentive pay to average stock prices is a better way to compensate executives. In this section, we investigate how to incorporate average stock prices in the payoffs of executive stock options. We focus on stock options instead of restricted stock because the former is the dominant form of incentive pay in practice. Restricted stock is also a special case of stock options when the strike price is zero, so our analysis is valid for restricted stock as well. We consider both traditional and indexed traditional options and discuss how average stock prices can be incorporated in their payoff structures and the subsequent impact on cost effectiveness and incentive effects.

3.1. Asian stock options

Most firms grant traditional stock options as opposed to non-traditional stock options (e.g., Johnson and Tian, 2000b). We therefore first examine how average stock prices can be incorporated into the payoff structure of traditional stock options. Suppose that a stock option is granted today with strike price \( K \) and maturity \( T \). If it is a traditional stock option, its payoff at maturity is:

\[
\text{Max}\{S_T - K, 0\}
\]

and the initial cost of granting the option to the firm is: \(^7\)

\[
V_{BS} = S_0 \exp(-qT)N(d_1) - K \exp(-rT)N(d_2)
\]

where

\[
d_1 = \frac{\log(S_0 / K) + (r - q + \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T},
\]

and \( N(x) \) is the cumulative probability function of the standard normal distribution.

On the other hand, if the stock option is based on the (geometric) average stock price over the life of the option, its payoff at maturity is given by:

\[
\text{Max}\{\hat{S}_T - K, 0\}.
\]

\(^7\) Early exercise, vesting and other features of executive stock options are ignored here in the analysis. These and other issues are discussed subsequently in Section 5.
This is an Asian option with a fixed strike price. The cost of granting the Asian stock option to the firm is:

\[ V_A = S_0 \exp(-\hat{q}T)N(d_1) - K \exp(-rT)N(d_2) \]  

\[ d_1 = \frac{\log(S_0 / K) + (r - \hat{q} + \frac{1}{2} \hat{\sigma}^2)T}{\hat{\sigma}\sqrt{T}}, \]

\[ d_2 = d_1 - \hat{\sigma}\sqrt{T}. \]

3.2. Indexed Asian stock options

Johnson and Tian (2000a) propose indexed traditional stock options that pay off only if the firm’s actual performance exceeds its expected performance relative to a benchmark index. Suppose that the benchmark is a stock market index and it follows a geometric Brownian motion:

\[ \frac{dI_t}{I_t} = (\mu_I - q_I)dt + \sigma_I dw_t \]  

where \( \mu_I \) is the expected rate of return on the market index, \( q_I \) is its dividend yield, \( \sigma_I \) is its volatility and \( w_t \) is a standard Brownian motion. The two Brownian motions driving the stock price and index level (\( z_t \) and \( w_t \)) are correlated, with \( \rho \) denoting the correlation coefficient.

Conditioning on the benchmark index, the expected stock price on a future date is given by:

\[ E(S_t | I_t) = S_0 \left( \frac{I_t}{I_0} \right)^\beta \exp(\eta^* t) \]  

where

\[ \eta^* = (\mu - q) - \beta(\mu_I - q_I) + \frac{1}{2} \sigma_I^2 \beta(1 - \beta). \]

\[ ^{8} \text{We do not consider floating-strike Asian options in this study because their payoff structure, } \max \{ S - \hat{S}, 0 \}, \text{ is not suited for incentive pay. Since risk-averse executives prefer receiving the average stock price, subtracting it from the option payoff is counterproductive. Likewise, the benefit of averaging cannot be achieved by granting a series of traditional stock options over time. Such a portfolio of traditional stock options resembles a floating-strike Asian option, much more than it does a fixed-strike Asian option that we adopt here in this study.} \]
\[
\beta = \frac{\rho \sigma}{\sigma_t}.
\]

If the firm is expected to earn zero excess return relative to the index, we should have
\[
\mu = r + \beta (\mu_t - r).
\]

In this case, Eq. (10) can be rewritten as:
\[
E(S_t | I_t) = S_0 \left( \frac{I_t}{I_0} \right)^\beta \exp(\eta t)
\]

where
\[
\eta = (r - q) - \beta (r - q_t) + \frac{1}{2} \sigma_t^2 \beta (1 - \beta).
\]

To have the option’s strike price indexed to the expected stock price, we should set the strike price as:
\[
H_t = K(I_t / I_0)^\beta \exp(\eta t).
\]

The payoff of the indexed traditional stock option at maturity is thus:
\[
\text{Max}\{S_T - H_T, 0\}.
\]

Note that both traditional and indexed traditional stock options have the same strike price \(K\) initially on the grant date. Unlike the traditional stock option, however, the strike price of the indexed traditional stock option is not constant but varies with the performance of the benchmark index (adjusted for the level of the stock’s systematic risk \(\beta\)). The cost of granting the indexed traditional stock option to the firm is given by the formula (see Johnson and Tian, 2000a):
\[
V_t = \exp(-qT)\left[ S_0 N(\hat{d}_1) - H_0 N(\hat{d}_2) \right]
\]

where
\[
d_1 = \frac{\log(S_0 / H_0) + \frac{1}{2} \sigma^2 (1 - \rho^2) T}{\sigma \sqrt{(1 - \rho^2) T}},
\]
\[
d_2 = d_1 - \sigma \sqrt{(1 - \rho^2) T}.
\]

Structuring option payoff on excess performance as opposed to total performance seems like an interesting way to address the criticism against excessive executive compensation via traditional stock options. By construction, roughly only half of the firms are likely to perform
better than expected and only executives in these firms will receive any payoff from their indexed traditional stock options. The remainder will see their indexed traditional stock options expiring out of the money. Due to a lower probability of expiring in the money, indexed traditional options are also criticized for being inefficient as it may cost the firm more to provide a given level of incentives if the firm chooses to grant indexed traditional stock options instead of traditional stock options (e.g., Hall and Murphy, 2003; Garvey and Milbourn, 2003). Not surprisingly, most executives prefer traditional stock options to indexed traditional stock options and virtually all firms grant traditional stock options to their top executives and other employees.

Is there a simple way to improve the effectiveness of indexed stock options? Incorporating the Asian payoff feature may just be the right prescription. Similar to the average stock price, we can define the geometric average index price as:

$$\hat{I}_t = \exp\left(\frac{1}{t} \int_0^t \log \hat{I}_u du\right)$$

which also follows a geometric Brownian motion:

$$\frac{d\hat{I}_t}{\hat{I}_t} = (\hat{\mu}_t - \hat{q}_t)dt + \hat{\sigma}_t d\hat{w}_t$$

where

$$\hat{\mu}_t = \frac{1}{2}(\mu_t + r) ,$$

$$\hat{q}_t = \frac{1}{2}(r + q_t + \frac{1}{6}\sigma_t^2) ,$$

$$\hat{\sigma}_t = \frac{\sigma_t}{\sqrt{3}} ,$$

and \( \hat{w}_t \) is a standard Brownian motion. The two Brownian motions driving the average stock price and average index level (\( \hat{z}_t \) and \( \hat{w}_t \)) have the same correlation coefficient (\( \rho \)) as the original two Brownian motions for the stock price and index level (\( z_t \) and \( w_t \)).

To incorporate average prices in the payoff structure of the indexed stock option, we can define the option payoff using both average stock price and average index level:

$$\text{Max}\{\hat{S}_T - \hat{H}_T, 0\}$$
where the strike price is based on the average stock price conditioning on the average index level:\(^9\)

\[
\hat{H}_t = K(\hat{I}_t / \hat{I}_0)^{\hat{\beta}} \exp(\hat{\eta}t) \quad (14)
\]

where

\[
\hat{\eta} = (r - \hat{q}) - \hat{\beta}(r - \hat{q}_t) + \frac{1}{2} \hat{\sigma}_t^2 \hat{\beta}(1 - \hat{\beta}),
\]

\[
\hat{\beta} = \frac{\rho \hat{\sigma}}{\hat{\sigma}_t}.
\]

It is straightforward to show that the strike price follows the geometric Brownian motion:

\[
\frac{d\hat{H}_t}{\hat{H}_t} = (r - \hat{q})dt + \rho \hat{\sigma}d\hat{\nu}_t \quad (15)
\]

We will refer to this new type of indexed options as \textit{indexed Asian stock options} whose payoffs are based on both the average stock price and the average index level. Following Johnson and Tian (2000a), the cost of the indexed Asian stock option to the firm is given by the formula:

\[
V_{IA} = \exp(-\hat{q}T)[S_0 N(d_1) - \hat{H}_0 N(d_2)] \quad (16)
\]

where

\[
d_1 = \frac{\log(S_0 / \hat{H}_0) + \frac{1}{2} \hat{\sigma}^2 (1 - \rho^2)T}{\hat{\sigma} \sqrt{(1 - \rho^2)T}},
\]

\[
d_2 = d_1 - \hat{\sigma} \sqrt{(1 - \rho^2)T}.
\]

3.3. \textit{Comparison of option costs to the firm}

By incorporating average stock prices into option payoffs, we have developed two new types of executive stock options: Asian stock options and indexed Asian stock options. How do these options compare with traditional and indexed traditional stock options? Since the average stock price has lower volatility and lower expected return than the stock price itself, it is quite clear that the Asian stock options should have lower market value than the corresponding

\(^9\) As in Johnson and Tian (2000a), the expected return on the stock is assumed to be determined by the systematic risk component (or equivalently the idiosyncratic risk is not priced): \(\mu = r + \beta(\mu_t - r)\).
traditional stock options do. It should thus cost the firm less to grant Asian and indexed Asian stock options. Numerical analysis in Table 1 confirms this intuition.

In Table 1, we report the market value (a proxy to the cost of the option grant to the firm) for all four types of options: traditional (denoted BS in the table), Asian, indexed traditional, and indexed Asian stock options. A market index such as the S&P 500 is assumed to be the benchmark for structuring indexed traditional and indexed Asian stock options. To perform the numerical analysis, we choose a base set of parameters that are consistent with those of a typical large cap stock in the U.S.: The initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, the return volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) varies from 0 to 0.75, the expected index return is 8%, the expected stock return varies from 4% to 10%, option maturity is 10 years, and strike prices vary from 80 to 120. The Capital Asset Pricing Model is assumed to hold and the expected return on the stock is determined by its systematic risk: $\mu = \beta (\mu_I - r)$.

As shown in Table 1, both Asian and indexed Asian stock options are indeed less expensive for the granting firm than the corresponding traditional and indexed traditional stock options. If the option is granted at the money (i.e., strike price = $100), the traditional stock option has a market value of $35.13. It thus costs the firm $35.13 per share to grant at-the-money traditional stock options. In comparison, it costs the firm only $15.51 per share to grant the corresponding Asian stock option or just 44.2% of the cost of granting traditional stock options. The cost of the indexed traditional stock option is also much lower than that of the traditional stock option, varying from $20.16 to $29.86 (or 57.4% to 85.0% of the cost of granting traditional stock options) depending on the correlation between stock and index returns. The indexed Asian stock option has the lowest market values, varying from $9.88 to $14.37 (or 28.1% to 40.9% of the cost of granting traditional stock options) depending on the correlation between stock and index returns.

The difference in market values varies with the strike price and generally becomes larger (smaller) when the option is out of the money (in the money). As discussed previously, the Asian stock option is worth 44.2% as much as the corresponding traditional stock option ($15.51 vs. $35.13) if the option is at the money. In comparison, when the strike price is raised to $120, the Asian stock option is worth only 35.9% as much as the corresponding traditional stock option.
($10.88 vs. $30.32). The reverse is true when the strike price is lowered to $80, with the Asian stock option now worth 53.7% as much as the corresponding traditional stock option ($22.05 vs. $41.04). For indexed traditional and indexed Asian stock options, there is a similar pattern of differences in valuation. The market value of the indexed Asian stock option is also substantially lower than the corresponding indexed traditional stock option, and the percentage difference is greater (smaller) for out-of-the-money (in-the-money) options.

Given the market values reported in Table 1, it is clearly important to adjust for differences in the cost of the options to the firm. In order to provide a proper comparison of incentive effects between these options, we must hold the total cost of the options constant. The number of options is calculated and adjusted for differences in option cost. Only then it is appropriate to analyze and compare the executive subjective valuation of the option grant and its incentive effects.

4. Certainty equivalent value and incentive effects

As demonstrated previously in Section 3, incorporating average stock prices in option payoffs can have a substantial impact on the option’s market value. Of course, the market value of the option only reflects the cost of the option to the granting firm but not necessarily its subjective value to the executive holding the option. The subjective value can be substantially lower due to risk aversion and hedging restrictions. In this section, we analyze the executive’s subjective valuation of four types of stock options and the incentive effects provided by these options. Since the optimal contract cannot be solved analytically when equity incentives are provided by stock options, we rely on certainty equivalent value from an expected utility model to perform the required analysis.

4.1. Certainty equivalent value

The executive is again assumed to be risk averse and has constant relative risk aversion (see Eq. (3)). The firm compensates the executive with a combination of fixed salary and stock options. Only one type of options (either traditional, Asian, indexed traditional or indexed Asian
options) is granted and the options cannot be exercised until maturity. The executive’s outside wealth is invested in the risk-free asset\(^{10}\) and the subjective value of the option is based on the expected terminal wealth at option maturity.

Let \(W_0\) be the executive’s initial wealth on grant date. While a \(\lambda\) fraction of the initial wealth is accounted by a stock option grant provided by the firm, the remainder is invested in the risk-free asset. The total number of options granted to the executive is:

\[
n = \frac{\lambda W_0}{V_0},
\]

where \(V_0\) is the cost of each option to the firm. The executive’s wealth at option maturity is thus:

\[
W_T = (1 - \lambda)W_0 \exp(rT) + nV_T,
\]

where \(V_T\) is option payoff at maturity. The exact form of the payoff depends on the type of options the executive has received. The ending wealth can be written in a more convenient format:

\[
W_T = \left[(1 - \lambda)\exp(rT) + \lambda \frac{V_T}{V_0}\right]W_0.
\]

The subjective value of the stock option to the executive is generally lower than the option’s market value, commonly measured as the certainty equivalent (CE) value that solves the following expected utility problem:

\[
u\{[(1 - \lambda)W_0 + n \cdot CE]\exp(rT)} = E[u(W_T)],
\]

where the expectation is taken under the actual probability measure (as opposed to the risk-neutral probability measure used for determining the cost of the stock option). In other words, the executive is indifferent between receiving the option grant and a cash payment of \(n \cdot CE\) on the grant date. Combining with the ending wealth in Eq. (17a) and utility function in Eq. (3), the expected utility equation can be simplified to:

\[
u\left(1 - \lambda\right) + \lambda \frac{CE}{V_0} = E\left[u\left(1 - \lambda\right) + \lambda \frac{V_T}{V_0} \exp(-rT)\right]
\]

Solving Eq. (18a), we find the subjective value of the stock option to the executive:

\(^{10}\) Note that our results are not materially affected if the executive’s outside investment opportunities are expanded to include the market portfolio (see robustness results in Fig. 9 of Section 5). The simplifying assumption is made to keep it more tractable without any loss of generality.
where \( u^{-1}(\cdot) \) is the inverse of the utility function.

As shown in Eq. (18b), the subjective value of the option (CE) can be stated as a fraction of its market value (i.e., a fraction of its cost to the granting firm). This fraction,

\[
\frac{CE}{V_0} = 1 - \frac{1 - u^{-1}\left(\mathbb{E}\left[u\left((1 - \lambda) + \frac{V_T}{V_0} \exp(-rT)\right)\right]\right)}{\lambda},
\]  

(18c)
is a function of the executive’s degree of risk aversion (\( \gamma \)), the fraction of wealth tied to stock options (\( \lambda \)), and the option payoff which depends on terms of the option grant and characteristics of stock returns (and the benchmark index returns if they are indexed stock options). Interestingly, the level of wealth (\( W_0 \)) does not factor into the determination of the option’s subjective value. It is the fraction of wealth tied to stock options (\( \lambda \)) that directly influences the option’s subjective value. This is not surprising since we assume the executive has constant relative risk aversion.

For risk-averse executives, the subjective value of the stock option can be substantially less than the cost of the option to the firm. More importantly, the discount may differ substantially across different types of stock options. To quantify the comparative differences in subjective values across different types of stock options, we tabulate the option’s subjective value as a fraction of its market value in Table 2. This fraction is 1 if the executive does not discount the option’s market value at all and is inversely related to the degree of risk aversion. The larger the discount, the smaller the subjective value as a fraction of its market value. We use the same base-case parameters as in Table 1 for the stock, index and option parameters. The executive’s coefficient of relative risk aversion (\( \gamma \)) varies from 2 to 4 and the fraction of wealth in stock options (\( \lambda \)) varies from 25% to 75%.

\[ \text{\ref{footnote:1}} \]

\[ \text{\ref{footnote:1}} \] To ensure robustness, we also consider a variety of other parameter values with the risk-free rate (\( r \)) ranging from 2% to 6%, market risk premium (\( \mu_I - r \)) ranging from 2% to 5% and stock return volatility (\( \sigma \)) ranging from 20% to 40%. All results remain qualitatively similar.
As expected, the executive discounts the option more if he or she is more risk averse (higher $\gamma$) and the weight in options is greater (larger $\lambda$). For instance, an at-the-money traditional stock option (reported in Panel A of Table 2) is only worth 9.6% of its market value to the executive if the risk aversion level is 4 and the weight in options is 75%. The subjective value as a fraction of market value rises to 87.3% if the risk aversion level drops to 2 and the weight in options is reduced to 25%. The variation in discount is thus quite large depending on the executive’s degree of risk aversion and weight in options.

More importantly, incorporating average stock prices in option payoffs can have a large impact on the option’s subjective value. For traditional stock options, averaging reduces the executive’s discount substantially if the option is in the money (Panel B), reduces it slightly if the option is at the money (Panel A), and increases it moderately if the option is out of the money (Panel C). This is quite clear by directly comparing the subjective values of traditional and Asian stock options. For indexed stock options, the effect of averaging is even stronger, consistently reducing the executive’s discount of the option value. In fact, the executive consistently value indexed Asian stock options higher than the corresponding indexed traditional stock options.

Results in Table 2 suggest that incorporating average stock prices may indeed improve the option’s cost effectiveness by reducing the executive’s discount of the option’s market value. The extent of the improvement becomes greater as the option’s strike price is reduced, particularly for an option that is already in the money. Does this mean that executives prefer receiving Asian or indexed Asian options as opposed to traditional or indexed traditional options? For that to be true, the executive must have a higher subjective value for Asian or indexed Asian options than for traditional or indexed traditional options when the cost of the stock options to the granting firm is held constant.

To answer this important question, we take another look at the results in Table 2 and provide a new interpretation of the findings. In Table 2, we report the option’s subjective value as a fraction of its market value for a given level of the executive’s degree of risk aversion ($\gamma$), total initial wealth ($W_0$), and portfolio weight in options ($\lambda$). As the total cost of the option grant to the firm ($\lambda W_0$) is held constant across all four types of options, the option’s subjective value as
a fraction of its market value (a proxy for the cost of the option grant to the firm) provides a valid comparison of the executive’s preferences for these options. A higher subjective value means that the executive has a greater expected utility of wealth or a larger certainty equivalent value for the option grant. Take the executive with a relative risk aversion of 2 and 50% of initial wealth in options. If the options are granted at the money, the subjective value of the option grant is 44.0%, 46.2%, 25.6% and 31.7% of its market value for traditional, Asian, indexed traditional and indexed Asian stock options, respectively. In all four cases, the executive discounts the market value of the option grant substantially but gives the Asian stock options the highest valuation (46.2%). Holding the cost of the option grant constant, the executive prefers receiving Asian stock options instead of the other three types of options. For the two types of indexed options, indexed Asian stock options are preferred to indexed traditional stock options.

Preference for Asian and indexed Asian options is not limited to this one particular case (i.e., at-the-money options for a risk-averse executive with a degree of risk aversion of 2 and 50% wealth in stock options). In fact, a risk-averse executive prefers receiving Asian stock options over the other three types of options as long as the options are granted at or in the money. In other words, the executive discounts the Asian stock options the least among the four types of options and thus prefer receiving Asian stock options, holding the cost of the options to the firm constant. If companies choose to grant out-of-the-money options (which is almost never done in practice), traditional options are valued slightly higher than Asian options by risk-averse executives. In this case, the discount from market value is so large for both types of options that it does not seem cost effective for firms to grant any options out of the money regardless of the option type. As a result, the slight advantage out-of-the-money traditional stock options have over out-of-the-money Asian stock options is likely a moot point in practice. For indexed options, the benefit of averaging is even stronger, with risk-averse executives preferring indexed Asian options over indexed traditional options in all cases reported in Table 2.

The benefit of averaging is further illustrated in Fig. 2 where we plot the executive’s subjective value as a fraction of market value across a wide range of moneyness (with the ratio of strike price over stock price varying from 0 to 2). The base-case parameters for stock and index returns are used and the executive has a degree of risk aversion of 3 and 50% of wealth in stock options. The advantages of Asian and indexed Asian options over traditional and indexed traditional options are quite clear in Fig. 2. If firms grant at-the-money options (as the
overwhelming majority of firms do in practice), the risk-averse executive clearly prefer receiving Asian (indexed Asian) options than the corresponding traditional (indexed traditional) options, with Asian options valued the highest and indexed traditional options valued the lowest. Granting options in the money improves the cost effectiveness of all four types of options since the discount shrinks as strike price or moneyness declines. More importantly, the advantage of Asian (indexed Asian) options over the corresponding traditional (indexed traditional) options increases as the options is even larger as the options become more in the money. The difference can be more than 20 percentage points if the options are sufficiently in the money.

Interestingly, averaging can also deflect the criticism that indexed options are less cost effective than traditional options, a common argument against the use of indexed options. As shown in Fig. 2, risk-averse executives prefer indexed Asian options over traditional options if they are sufficiently in the money (e.g., moneyness ≤ 0.65). Of course, traditional stock options are still preferred over indexed Asian options if both are granted at the money. The gap in valuation is, however, smaller than it is between traditional and indexed traditional options. These results imply that indexed options should not be dismissed as a potential choice of executive stock options, especially those with the Asian payoff structure. Firms should not automatically choose traditional stock options for their stock option plans. The optimal choice should be based on an appropriate tradeoff between cost effectiveness and incentive effects.

In addition, we should also rethink the common practice of granting options at the money. U.S. firms have rarely granted in-the-money options in the past due to their unfavorable accounting treatment. Until recently, U.S. firms did not have to expense any cost associated with stock option grants as long as they were granted at or out of the money. In contrast, the difference between stock price and strike price had to be expensed (i.e., deducted against earnings) in the fiscal year the options were granted if they were granted in the money. Such preference for at-the-money options is no longer warranted as the implementation of the Financial Accounting Standard Board’s Statement of Financial Accounting Standard No. 123R now requires all U.S. companies to expense their stock option grants at cost, regardless of moneyness. It is thus time to re-evaluate the common practice of granting at-the-money options. Some companies (e.g., Microsoft and other technology firms) have already started granting more
restricted stock (which is equivalent to in-the-money options with a zero strike price) and simultaneously reducing stock option grants. Such changes are in fact more widespread and have led to a sizeable shift away from traditional stock options to restricted stock in executive compensation (e.g., Feng and Tian, 2009). This shift is certainly consistent with the results in Table 2 and Fig. 2 which show that the cost effectiveness of stock options improves as the strike price declines.

4.2. Incentives to increase stock price

Of course, being more cost effective is not enough. Holding the cost of the option grant to the firm constant, do Asian (indexed Asian) stock options provide stronger incentives to increase stock price than traditional (indexed traditional) stock options? How do in-the-money options compare with at-the-money or out-of-the-money options? To answer these questions, we characterize and analyze managerial incentives provided by stock options following standard approach in the literature (e.g., Jensen and Murphy, 1990).

To measure incentives to increase stock price provided by stock options, we typically use aggregate option delta or its variants such as Jensen and Murphy’s (1990) pay-performance sensitivity (PPS). Here, we adopt a slightly modified PPS measure that is based on percentage changes. It is defined as the ratio of the percentage change in the executive’s total wealth to the percentage change in stock price. A ratio of 1 thus has the interpretation that a 1% increase in stock price will lead to a 1% increase in the executive’s total wealth. This modified PPS measure is equivalent to Jensen and Murphy’s (1990) PPS measure but rescaled to remove any reference to monetary units in the ratio. The rescaled PPS allows for easy comparison of incentive strengths across different types of options.

We begin with the total certainty equivalent value (TCE) of the executive’s entire portfolio (consisting of risk-free assets and stock options) on grant date:

$$TCE = (1 - \lambda)W_0 + n \cdot CE,$$

or equivalently:

$$TCE = \left[ (1 - \lambda) + \lambda \frac{CE}{V_0} \right] W_0.$$

To determine our PPS measure, we first calculate the delta of the option’s subjective value by taking partial derivative with respect to stock price on both sides of Eq. (18a):
\[ \frac{\partial (CE)}{\partial S} = \frac{E \left[ u' \left( 1 - \lambda + \frac{V_r}{V_0} \exp(-rT) \right) \frac{\partial V_r}{\partial S} \right] \exp(-rT)}{u' \left( 1 - \lambda + \frac{CE}{V_0} \right)} . \]

The PPS measure is thus:

\[ \frac{\partial (TCE)}{\partial S} \frac{S}{TCE} = \frac{E \left[ u' \left( 1 - \lambda + \frac{V_r}{V_0} \exp(-rT) \right) \frac{\partial V_r}{\partial S} \right] \lambda S \exp(-rT)}{u' \left( 1 - \lambda + \frac{CE}{V_0} \right)} \left\{ (1 - \lambda) V_0 + \lambda CE \right\} . \] (19)

Results of a numerical analysis of the PPS incentives are reported in Table 3 and Fig. 3 using the base-case parameters for stock and index returns. As the option cost per share varies substantially across the four types of options (see Table 1), we hold the total cost of the option grant constant before comparing their incentives in order to provide a valid comparison across these options. The number of stock options the executive receives \((n)\) thus depends on the type of options granted (traditional, Asian, indexed traditional, or indexed Asian stock options) and the option’s moneyness (at, in or out of the money). Once the number of options is determined, it is held constant in the PPS calculation. For each option type and moneyness, we then use Eq. (19) to calculate the PPS measure.

In Fig. 3, we plot the PPS incentives against moneyness (strike price over stock price) for an executive whose coefficient of risk aversion is 3 and stock options account for 50% of the initial wealth. As shown in Fig. 3, the incentives (PPS) provided by the option grant differ substantially across option types and moneyness. The Asian and indexed Asian options provide the strongest incentives to increase stock price. Over a large range of moneyness levels (between 0.2 and 1), the PPS of these option grants generally varies between 0.5 to 0.75 (with a 1% increase in stock price leading to between 0.5% and 0.75% gain in the executive’s wealth). The level of incentives can be as high as 0.72 for Asian options and 0.78 for indexed Asian options. In comparison, the highest level of incentives provided by traditional and indexed traditional options is only 0.41 and 0.43, respectively. The incentives provided by Asian and indexed Asian options are thus nearly twice as high as those provided by traditional and indexed traditional options.
Are the differences in incentive strength illustrated in Fig. 3 more general or just for that special case? To answer this important question, we perform a more extensive analysis of PPS incentives for executives with both moderate and high levels of risk aversion ($\gamma$) and a range of portfolio weight in stock options ($\lambda$) and report the results in Table 3. Similar to the pattern illustrated in Fig. 3, the incentives provided by Asian and indexed Asian options are consistently larger than those provided by traditional and indexed traditional options across wide variations in risk aversion level and portfolio weight. Take the case of an executive with a moderate level of risk aversion ($\gamma = 2$) and a moderate level of wealth in options ($\lambda = 25\%$). The maximum level of incentives (PPS) provided by traditional options is 0.326 when the options are granted with a $95 strike price. In comparison, the maximum level of incentives provided by Asian options is 0.553 (at strike price $100$) which is 69\% greater. Similarly, the maximum level of incentives provided by indexed traditional options is 0.319 (at $55$ strike price) compared to 0.529 provided by indexed Asian options (at $85$ strike price). The latter is 66\% greater than the former. At the other extreme for an executive with a high level of risk aversion ($\gamma = 4$) and a high level of wealth in options ($\lambda = 75\%$), the maximum level of incentives provided by traditional options is 0.516 (at $20$ strike price) compared to 0.966 by Asian options (at $35$ strike price). The latter is 87\% higher than the former, nearly doubling the incentive strength. For indexed traditional and indexed Asian options, the difference is slightly higher at 90\%, with indexed traditional options providing a maximum level of incentives of 0.559 (at $15$ strike price) compared to 1.061 (at $35$ strike price) provided by indexed Asian options. We thus conclude that Asian and Asian indexed options provide much stronger incentives for executives to increase stock price than traditional and indexed traditional options. More importantly, Asian (indexed Asian) options completely dominate traditional (indexed traditional) options as a possible choice of incentive compensation in the sense that the former always provides larger incentives to increase stock price (PPS) than the latter at all moneyness levels (holding the total cost of the option grant to the firm constant). This is quite clear from Fig. 3 and Table 3.
Another interesting finding from the results in Table 3 and Fig. 3 is that companies are also better off by granting options in the money than either at or out of the money. The level of incentives (PPS) typically reaches the maximum in the money and then monotonically declines as strike price increases (moving the option into the out-of-the-money region or making it more out of the money). This is true for all types of options, different levels of risk aversion and different portfolio weights in options. The common practice of granting options at the money is thus suboptimal. Companies should consider granting options in the money, with the optimal choice of moneyness somewhere between 0 and 1. The exact level of moneyness that provides the maximum incentives may depend on the executive’s level of risk aversion and percentage of wealth in options and characteristics of firm and index returns.

In addition, Asian options do not dominate indexed Asian options as a potential choice of incentive pay. As the strike price increases from 0, indexed Asian options initially dominate Asian options, providing stronger incentives to increase stock price (PPS). This is quite clear from Fig. 3, as indicated by the top two lines. These two lines then cross over roughly at a moneyness level just below 0.8, with both types of Asian options providing identical levels of incentives at that point. After the crossover, Asian options dominate indexed Asian options, providing stronger incentives to increase stock price as moneyness (or strike price) increases further. A similar pattern is observed between traditional and indexed traditional options.

4.3. Incentives to increase risk

The results in Fig. 3 and Table 3 clearly show that Asian (indexed Asian) stock options provide much stronger incentives to increase stock price than traditional (indexed traditional) stock options do, holding the cost of the option grant to the firm constant. This is an important finding and supports the optimal contracting results provided in Section 2 on the benefit of linking incentive pay to average stock prices. Does this gain in incentive strength come about with any unfavorable changes in risk incentives? We next analyze and compare risk incentives provided by these four types of stock options.

Unlike incentives to increase stock price, there are several different measures for risk incentives. This is because total risk can be decomposed into systematic and idiosyncratic components:

\[ \sigma^2 = \beta^2 \sigma_i^2 + \varepsilon^2, \]
where the systematic component, stated in standard deviation, is
\[ \beta \sigma_i = \rho \sigma \]
and the idiosyncratic component, also stated in standard deviation, is
\[ \epsilon = \sqrt{1 - \rho^2} \sigma \].

Do stock options provide incentives to change the firm’s systematic risk (e.g., moving to another industry), idiosyncratic risk (e.g., changing tactics in union contract negotiations), or a combination of both (e.g., increasing leverage)? While changes in systematic risk affect the stock’s expected return, changes in idiosyncratic risk do not. This distinction is important because systematic risk is priced by diversified investors while idiosyncratic risk is not. As a result, changes in systematic risk lead to changes in the stock’s expected returns. In comparison, changes in idiosyncratic risk do not affect the stock’s expected returns. This means that a risk-averse executive may prefer an increase in systematic risk if the subsequent increase in expected return is more than sufficient to compensate for the increase in uncertainty. In contrast, the executive is unlikely to take any action to increase idiosyncratic risk because it simply raises the level of uncertainty without any compensation in higher returns. It is thus important to separately investigate the incentives to increase systematic risk and the incentives to increase idiosyncratic risk.

We first examine incentives to increase systematic risk, defined as the ratio of the percentage change in the certainty equivalent value of total wealth to the percentage change in systematic risk. By taking partial derivative with respect to \( \beta \) on both sides of Eq. (18b), we characterize the sensitivity of the stock option’s subjective value to changes in systematic risk:

\[
\frac{\partial (CE)}{\partial \beta} = \frac{E\left[u\left(1 - \lambda + \lambda \frac{V_T}{V_0} \exp(-rT) \frac{\partial V_T}{\partial \beta}\right) \exp(-rT)\right]}{u\left(1 - \lambda + \lambda \frac{CE}{V_0}\right)}.
\]

The incentive to increase systematic risk is thus:

\[12\] Although the discussion on risk incentives here is based on the classical CAPM definition of systematic and idiosyncratic risks, the analysis remains valid under any setting that distinguishes between changes in risk that do and do not lead to changes in expected return.
\[
\frac{\partial (\text{TCE})}{\partial \beta} = \frac{\mathbb{E}\left[ u^T \left( 1 - \lambda + \frac{\lambda}{V_0} \exp(-rT) \right) \frac{\partial V_T}{\partial \beta} \right]}{\mathbb{E}\left[ u^T \left( 1 - \lambda + \frac{\lambda}{V_0} CE \right) \right]} \cdot \frac{\lambda \beta \exp(-rT)}{(1 - \lambda)V_0 + \lambda CE}
\]

To isolate the impact of systematic risk, we hold the idiosyncratic risk constant in the calculation of incentives to increase systematic risk in Eq. (20). In other words, we hold \( \sigma = \sqrt{1 - \rho^2} \sigma \) constant while allowing \( \beta \) to change. Of course, a change in systematic risk will necessarily lead to a corresponding change in the stock’s expected return (\( \mu \)), its correlation with the index return (\( \rho \)), and its total risk (\( \sigma \)). To calculate the incentives to increase systematic risk using Eq. (20), these implicit relationships must be maintained in order to determine the correct measure of risk incentives.

Incentives to increase systematic risk are illustrated in Fig. 4 using the same base-case parameters for stock and index returns. Again, we hold the total cost of the option grant to the firm constant in order to provide a proper comparison across the four types of options. The executive’s coefficient of risk aversion is 3 and stock options account for 50% of the initial wealth. As shown in Fig. 4, stock options provide little incentive to increase or decrease systematic risk. This is particularly true for indexed and indexed Asian options, with the incentives close to zero at nearly all moneyness levels. In comparison, the incentives to increase systematic risk are slightly elevated for traditional and Asian stock options. They provide some incentives to decrease (increase) systematic risk when the options are granted in the money (out of the money). Nevertheless, the incentives to change systematic risk are generally quite weak for all four types of stock options. A 1% change (increase or decrease) in systematic risk leads to less than 0.08% change in the executive total wealth. Though not reported, we show that the pattern of weak incentives to change systematic risk remains true over a wide range of parameter values. We thus conclude that stock options create little incentives to change systematic risk, especially indexed and indexed Asian stock options. The increase in expected return is just enough to compensate for the increase in systematic risk (while holding the idiosyncratic risk fixed).
What about incentives to change idiosyncratic risk? As discussed previously, stock options are unlikely to create any incentives to increase idiosyncratic risk but likely to create strong incentives to decrease idiosyncratic risk. This is because changes in idiosyncratic risk do not have any effect on expected stock returns. Holding the expected stock return constant, risk-averse executives prefer less uncertainty and thus lower idiosyncratic risk (while holding systematic risk constant). To find out if this line of reasoning is correct, we next analyze incentives to increase idiosyncratic risk. As in the case of systematic risk, incentives to increase idiosyncratic risk can be defined as the ratio of the percentage change in the certainty equivalent value of the executive’s total wealth to the percentage change in idiosyncratic risk. Technically, this risk incentive is calculated as:

\[
\frac{\partial(TCE)}{\partial \epsilon} \cdot \frac{\epsilon}{TCE} = \frac{E\left\{u'\left[\left(1-\lambda\right)+\frac{\lambda V_u}{V_0}\exp(-rT)\right] \frac{\partial V_T}{\partial \epsilon}\right\}}{u'\left[\left(1-\lambda\right)+\frac{\lambda CE}{V_0}\right]} \cdot \frac{\lambda \epsilon \exp(-rT)}{(1-\lambda)V_0 + \lambda CE}
\]

To isolate the impact of idiosyncratic risk, we hold the systematic risk constant in the calculation of incentives to increase idiosyncratic risk in Eq. (21). In other words, we hold \( \beta \) constant while allowing \( \epsilon = \sqrt{1-\rho^2 \sigma} \) to change. By holding the systematic risk constant, the stock’s expected return (\( \mu \)) will remain unchanged. However, both the stock’s total risk (\( \sigma \)) and its correlation with the index return (\( \rho \)) will be different. To calculate the incentives to increase idiosyncratic risk using Eq. (21), these changes must be incorporated in order to determine the correct measure of risk incentives.

Incentives to increase idiosyncratic risk are illustrated in Fig. 5 using the same base-case parameters for stock and index returns. Again, we hold the total cost of the option grant to the firm constant in order to provide an appropriate comparison across the four types of options. The executive’s coefficient of risk aversion is 3 and stock options account for 50% of the initial wealth, similar to the scenario depicted in Fig. 4. As shown in Fig. 5, all four types of stock options create strong incentives to reduce idiosyncratic risk if the options are granted in the money (i.e., moneyness < 1). This is consistent with the intuition that risk-averse executives do not like changes that increase uncertainty without any benefit of higher expected returns. When the options are already in the money, lowering the idiosyncratic risk helps protect the expected positive option payoff which is beneficial to the risk-averse executive. As the option’s strike
price increases, especially past the at-the-money threshold (i.e., moneyness = 1), the incentives to
decrease idiosyncratic weakens considerably. In fact, the risk incentives change direction in
some cases (e.g., indexed Asian options) and turn into moderate risk-seeking ones. Nevertheless,
the incentive to increase idiosyncratic risk is quite weak, with a 1% increase in idiosyncratic risk
leading to no more than 0.09% increase in the certainty equivalent value of the executive’s total
wealth. The results in Fig. 5 thus suggest that stock options generally create incentives to reduce
idiosyncratic risk and little or no incentives to increase idiosyncratic risk. Though not reported,
we show that the pattern of incentives to change idiosyncratic risk remains qualitatively similar
for other parameter values.

Finally, we characterize incentives to increase total risk while holding the correlation
between the stock and index returns constant. In the previous analysis of risk incentives (as
illustrated in Figs. 4 and 5), both total risk ($\sigma$) and correlation ($\rho$) change simultaneously in order
to keep either systematic risk or idiosyncratic risk constant. Although the analysis helps us
isolate the impact of systematic risk from the impact of idiosyncratic risk and vice versa, the
required simultaneous changes (for both total risk and correlation) are more difficult to interpret
empirically. While changes in operational leverage or financial leverage can lead to a change in
total risk, the correlation with a market index is harder to change and may require a more drastic
overhaul in business structure (such as investments in other industries through mergers and
acquisitions or divestitures through spinoffs or curve-outs). It is thus empirically meaningful to
investigate incentives to change total risk while keeping the correlation constant.

To determine the incentives to increase total risk, we first calculate the vega of the
option’s subjective value by taking partial derivative with respect to $\sigma$ on both sides of Eq. (18b):

$$\frac{\partial (CE)}{\partial \sigma} = \left. \frac{E \left[ u' \left( (1 - \lambda) + \frac{\lambda V_T}{V_0} \exp(-rT) \right) \frac{\partial V_T}{\partial \sigma} \right] \exp(-rT)}{u' \left( (1 - \lambda) + \frac{\lambda CE}{V_0} \right)} \right|_{\sigma} .$$

The incentive to increase total risk, defined as the ratio of the percentage change in certainty
equivalent value of total wealth to the percentage change in total risk, is thus:
\[ \frac{\partial (TCE)}{\partial \sigma} \cdot \frac{\sigma}{TCE} = \frac{E\left\{ u'\left[ (1-\lambda) + \frac{\lambda V_r V_0}{\exp(-rT)} \right] \frac{\partial V_r}{\partial \sigma} \right\} \cdot \lambda \sigma \exp(-rT)}{u'\left[ (1-\lambda) + \frac{\lambda CE}{V_0} \right] (1-\lambda)V_0 + \lambda CE} \]  

Note that if the correlation is held constant, any change in total risk will lead to a proportional change in both systematic risk (\(\beta\)) and idiosyncratic risk (\(\epsilon\)). The change in systematic risk also leads to a corresponding change in the stock’s expected return (\(\mu\)). To calculate the incentives to increase total risk using Eq. (22), these changes must be incorporated in order to determine the correct measure of risk incentives.

The incentives to increase total risk are illustrated in Fig. 6 using the same base-case parameters for stock and index returns. Again, we hold the total cost of the option grant to the firm constant in order to provide an appropriate comparison across the four types of options. The executive’s coefficient of risk aversion is 3 and stock options account for 50% of the initial wealth. As shown in Fig. 6, stock options provide little or no incentive to increase total risk (while holding the correlation between stock and index returns constant). Instead, they appear to provide strong incentives to decrease total risk if they are granted in the money. This is true for all four types of options and across a wide range of strike prices. This is in contrast to what is predicted by option pricing theory which suggests that an increase in total risk leads to an increase in the option’s market value. Unlike a typical investor who can hedge the uncertainty in option payoffs by trading the underlying stock, the risk-averse executive cannot undertake such hedging activities and thus does not like any increase in uncertainty unless a sufficient increase in expected payoff is provided to compensate for the rise in uncertainty. For in-the-money options, especially deep in-the-money options, the probability of a positive payoff is very high. The executive prefers to reduce uncertainty (other things being equal) in order to maintain or even increase the already high probability of a positive payoff. This is consistent with previous research that also finds that options can provide incentives to reduce total risk (e.g., Lambert et al., 1991; Carpenter, 2000; Ju et al., 2003; Ross, 2004). To encourage executives to take on more risky (and positive NPV) investment projects, companies may have to grant options at or out of the money, especially if executives already have outstanding options that are deep in the money.
In sum, incorporating average stock prices in option payoffs appears to have little effect on stock options’ risk incentives. For both traditional and indexed options, it only slightly elevates the incentive to reduce risk (systematic or idiosyncratic) when the options are in the money and slightly elevates the incentives to increase risk (systematic or idiosyncratic) when they are out of the money. This is an important finding as it suggests that incorporating average stock prices into option payoffs improves the cost effectiveness and strengthens incentives to increase stock price for both traditional and indexed options without any inadvertent impact on risk incentives. The benefit of incorporating average stock prices in option payoffs is thus quite clear. These results are not surprising given the optimal contract analysis of linking incentive pay to average stock prices in Section 2.

5. Robustness, implementation and other practical issues

For any company considering adopting Asian stock option plans, an immediate implementation issue is how to calculate average stock prices. Our construction and analysis of Asian stock options are based on geometric average prices calculated continuously over the life of the option. In practice, it is more convenient to use average prices calculated at discrete frequencies (such as daily, weekly or monthly intervals). With discrete averaging, the geometric average stock price over the time period \([0, T]\) is calculated as:

\[
\hat{S}_n = \left( \prod_{i=1}^{n} S_i \right)^{1/n}
\]

where the period \([0, T]\) is divided into \(n\) equal intervals of length \(\Delta t\), and \(S_i\) is the stock price at time \(t = i\Delta t\).

How does discrete averaging affect the value and incentive effect of Asian stock options? Not much if daily or weekly averaging is used. From a well-known result in Turnbull and Wakeman (1991) (see their Eqs. (17a) and (17b)), it is straightforward to show that the discrete average stock price has the following mean, dividend yield, and volatility:

\[
\hat{\mu}_n = \left[ r + \frac{1}{2} (\mu - r) \left( 1 + \frac{1}{n} \right) \right] T,
\]
\[
\hat{q}_n = \left[ r - \frac{1}{2} (r - q - \frac{1}{2} \sigma^2) \left( 1 + \frac{1}{n} \right) - \frac{1}{2} \hat{\sigma}_n \right] T,
\]
\[
\hat{\sigma}_n = \sqrt{\left( 1 + \frac{1}{n} \right) \left( 1 + \frac{1}{2n} \right) \sigma^2} \sqrt{T}.
\]

As expected, the mean, dividend yield and volatility of the discretely averaged stock price converge to their respective limit of the continuously averaged stock price as the number of averaging periods \( (n) \) increases. For a ten-year option with daily or weekly averaging, discrete averaging has very little impact on the mean, dividend yield and volatility. With weekly averaging, for example, the average stock price over the life of the option is calculated from 520 weekly stock prices and \( 1 + \frac{1}{n} \approx 1.0019 \) and \( \sqrt{\left( 1 + \frac{1}{n} \right) \left( 1 + \frac{1}{2n} \right)} \approx 1.0014 \). With the base-case parameters for stock returns \( (\mu = 8\%, \ r = 4\%, \ q = 2\%, \ \text{and} \ \sigma = 30\%) \), the continuously averaged stock return has an annualized mean of 6\%, (pseudo) dividend yield of 3.75\% and volatility of 17.32\%. With weekly averaging, these numbers change slightly to 6.04\%, 3.73\%, and 17.57\%, respectively. The effect is even smaller with daily averaging. Not surprisingly, untabulated results show that the value and incentive effects of Asian and indexed Asian options are not materially affected by discrete averaging (either daily or weekly) and their comparative advantages over traditional and indexed traditional options remain.

Another practical concern is the implementation of indexing. Both indexed traditional and indexed Asian options require the estimation of three unknown parameters – stock return volatility, index return volatility and correlation between stock and index returns. In comparison, traditional and Asian options only require the estimation of a single parameter – stock return volatility. As companies are already familiar with the estimation of stock return volatility due to recent accounting changes regarding option expensing, estimating index return volatility should not present much difficulty or require much incremental effort. The real concern lies with the estimation of correlation or beta, particularly for thinly traded stocks or companies with little history of trading (e.g., recent IPOs). One possible solution is to benchmark directly against an industry, sector or market index that roughly mimics the performance of the firm’s stock without any adjustment for systematic risk (equivalent to assuming a beta of 1). For example, we can choose the Nasdaq 100 index for a typical technology firm, the S&P 500 index for a typical large
cap stock, or the S&P 600 index for a typical small cap stock. With an appropriately chosen index, we can then assume the beta of the stock is 1 and implement the indexing without estimating the correlation. This simplification is likely to introduce potential benchmarking errors (as the firm’s beta is likely different from 1) but is probably a reasonable compromise for most companies.

The analysis thus far has ignored the possibility of early exercise of stock options. Most stock options are granted with a vesting period that may vary from one to four years. Once the options are vested, the executives are free to exercise them at any time prior to maturity. These stock options effectively become American options upon vesting. How does early exercise affect the value and incentive effects of Asian and indexed Asian options? In Fig. 7, we plot the executive’s subjective value of traditional and Asian stock options as a fraction of market value across a wide range of moneyness from 0 to 2. Both European and American versions of the options are illustrated in the figure. The base-case parameters for stock and index returns are used and the executive has a degree of risk aversion of 3 and 50% of the initial wealth in stock options. A 400-period binomial model is used to calculate the value and incentive effects of American options. As shown in Fig. 7, American options are more cost effective than European options. In other words, the executive’s subjective value of the option as a fraction of its market value is higher for the American option than the corresponding European option. This is true for both traditional and Asian options. This is not surprising since early exercise provides an opportunity for the risk-averse executive to convert non-tradable/undiversified holdings in stock options into traded/diversified securities and hence eliminate the discount from market value earlier than if the options are European. More interestingly, in-the-money Asian options remain more cost effective than the corresponding traditional options, similar to their European counterparts do.

Similarly, we plot the incentive to increase stock price (PPS) in Fig. 8 for traditional and Asian options (both European and American versions of the options). It is clear that American options provide much stronger incentive to increase stock price than the corresponding European options. This is true for both traditional and Asian options. More importantly, American Asian
options provide stronger incentives to increase stock price than American traditional options do. The early exercise feature thus does not change the advantage Asian options have in incentive strength over traditional options.

Untabulated results also show that the findings in Figs. 7 and 8 apply to indexed traditional and indexed Asian options as well. The American versions of the indexed traditional and indexed Asian options are more cost effective and provide stronger incentive to increase stock price than the corresponding European versions of the same options. More importantly, the early exercise feature does not change the advantage indexed Asian options have in cost effectiveness and incentive effects over indexed traditional options. Our analysis and findings in Section 4 are thus not affected by early exercise.

In analyzing the value and incentive effects of Asian stock options, we have made the simplifying assumption that the executive’s outside wealth is invested entirely in the risk-free asset. This assumption is clearly too restrictive as most investors prefer to have some exposure to the stock market. To ensure that our results are not driven by the risk-free investment assumption, we expand the executive’s outside investment opportunities to include the market portfolio. The certainty equivalent value of the executive’s stock options is then recalculated to account for this expanded investment opportunity set. Results in Tables 2 and 3 and Figs. 2-6 are then reproduced to verify the robustness of our findings. Although the revised results are slightly different quantitatively, they remain qualitatively similar and our findings regarding the advantages of Asian over traditional stock options are still valid. For example, Fig. 9 illustrates the incentives to increase stock price when the executive’s outside investment opportunities are expanded to include the market portfolio. We use the same base-case parameters as in Fig. 3 where the same incentives are calculated under the assumption that the executive’s outside wealth is invested entirely in the risk-free asset. In Fig. 9, the curves labeled BS_m and Asian_m are incentives to increase stock price for traditional and Asian stock options, respectively, under the expanded investment opportunity assumption. The curves labeled BS_r and Asian_r are the

---

13 For the valuation of indexed and Asian indexed stock options, we adopt the bivariate binomial tree model developed by Boyle (1988) and Boyle et al. (1989).
corresponding incentives under the risk-free investment assumption (identical to the ones illustrated in Fig. 3). As shown in Fig. 9, the outside investment assumption does appear to affect the incentives quantitatively, with the risk-free investment assumption shifting the plot upward and slightly to the right. Under the risk-free investment assumption, the incentive strength is thus overestimated and the optimal strike price is shifted to a higher level. This is true for both traditional and Asian stock options. Nevertheless, the advantages of Asian over traditional stock options remain unchanged, with Asian stock options providing much strong incentives than traditional stock options do. Even the pattern of differences (between traditional and Asian stock options) remains virtually the same under both investment assumptions. Our findings are thus robust to the outside investment assumption.

Another potential concern is the problem of diminishing incentive effect as Asian options get closer to expiration. As averaging is performed over the life of the option, much of the average stock price is already determined when the option is near its maturity. Even a large stock price swing near the option’s maturity may not have much impact on the Asian option’s payoff. This means that companies should not provide mega grants such as those granted by Apple to its CEO Steve Jobs.\textsuperscript{14} With such mega grants, executives are provided with an unusually large option grant in one year and then little or no option grant in many subsequent years. For Asian options (and traditional stock option for that matter), it is better to spread the option grants over time, providing a smaller grant each year but allocating the options over many years. This way, the executives always have a substantial portion of their option portfolio in newly granted or recently granted options. This multi-year granting policy should alleviate much of the diminishing incentive effect problem of Asian options.\textsuperscript{15}

\textsuperscript{14} Apple gave its CEO Steve Jobs a mega grant of 20 million options in 2000 and another mega grant of 7.5 million options in 2002.

\textsuperscript{15} Another possible solution is to use moving window Asian options, with averaging over a fixed (say, two-year) but moving window. The cost of this alternative is computational complexity which typically requires Monte Carlo simulation for valuation. We may have trouble persuading corporate accountants to go with this approach.
6. Conclusions

In this paper, we argue that the common practice of granting traditional stock options is suboptimal. In particular, traditional stock options are not cost effective and do not provide sufficient or the right type of incentives to executives. Firms are better off linking incentive pay to average stock prices (e.g., by granting Asian stock options) as opposed to stock prices. Averaging reduces volatility substantially (by more than 42%) and provides a much better risk-return tradeoff for risk-averse executives. Under conditions mimicking a typical corporate contracting environment, we show that Asian stock options are more cost effective (i.e., discounted less from its market value by risk-averse executives) than traditional stock options are. They also provide much stronger incentives to increase stock price (i.e., higher pay-performance sensitivity) than traditional stock options do, holding the cost of the option grant to the firm constant.

More importantly, the stronger pay-performance sensitivity is achieved without any material effect on the option grant’s risk incentives. Incorporating average stock prices into option payoffs has little effect on the option grant’s incentives to change either systematic or idiosyncratic risk. This is an interesting result as there is no consensus in the literature on whether stock options create too much or too little incentives to increase risk. By increasing pay-performance sensitivity without any material change in risk incentives, Asian stock options provide a substantial jump in managerial incentives without any unintended consequences on managerial risk-taking behavior.

Another benefit of Asian stock options is that averaging also makes it harder for executives to profit from opportunistic manipulation of stock prices. This is because the payoff of traditional stock options is based on stock price at a single point in time while the payoff of Asian stock options is based on the average stock price over the option grant’s life. It is much more difficult to manipulate the stock price over the life of the option than over a single day. Other fraudulent behaviors that do not involve manipulation of stock prices such as option backdating can be circumvented by regulatory changes requiring more timely corporate disclosure (such as the Sarbanes-Oxley Act of 2002) or by adopting more rigid granting policies such as fixed-cost stock option plans as opposed to fixed-shares stock option plans. As the total market value of the option grant is predetermined in a fixed-cost stock option plan, option backdating neither changes the cost of the option grant to the firm nor makes it more beneficial.
to the executives. The number of options is adjusted downward to reflect the change in option value if it is backdated for a lower strike price.
References


Table 1
Comparison of market values for four types of executive stock options

This table compares the costs of the four types of executive stock options (i.e., market values) to the granting firm, including traditional stock options, Asian stock options, indexed stock options, and Asian indexed stock options. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) varies from 0 to 0.75, the expected index return is 8%, the expected stock return varies from 4% to 10%, option maturity is 10 years, and strike prices vary from 80 to 120.

<table>
<thead>
<tr>
<th>Strike price</th>
<th>Corr. ($\rho$)</th>
<th>Market value</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BS</td>
<td>Asian</td>
<td>Indexed</td>
</tr>
<tr>
<td>80</td>
<td>0.75</td>
<td>41.04</td>
<td>22.05</td>
<td>27.43</td>
</tr>
<tr>
<td>80</td>
<td>0.50</td>
<td>41.04</td>
<td>22.05</td>
<td>32.49</td>
</tr>
<tr>
<td>80</td>
<td>0.25</td>
<td>41.04</td>
<td>22.05</td>
<td>34.98</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
<td>41.04</td>
<td>22.05</td>
<td>35.75</td>
</tr>
<tr>
<td>100</td>
<td>0.75</td>
<td>35.13</td>
<td>15.51</td>
<td>20.16</td>
</tr>
<tr>
<td>100</td>
<td>0.50</td>
<td>35.13</td>
<td>15.51</td>
<td>26.10</td>
</tr>
<tr>
<td>100</td>
<td>0.25</td>
<td>35.13</td>
<td>15.51</td>
<td>28.98</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>35.13</td>
<td>15.51</td>
<td>29.86</td>
</tr>
<tr>
<td>120</td>
<td>0.75</td>
<td>30.32</td>
<td>10.88</td>
<td>14.89</td>
</tr>
<tr>
<td>120</td>
<td>0.50</td>
<td>30.32</td>
<td>10.88</td>
<td>21.18</td>
</tr>
<tr>
<td>120</td>
<td>0.25</td>
<td>30.32</td>
<td>10.88</td>
<td>24.27</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
<td>30.32</td>
<td>10.88</td>
<td>25.22</td>
</tr>
</tbody>
</table>
Table 2
The executive’s subjective value of the stock option as a percentage of market value

This table reports the executive’s subjective value of the stock option as a percentage of the option’s market value. Comparison is made for four types of executive stock options: traditional stock options, Asian stock options, indexed stock options, and Asian indexed stock options. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index and stock returns are both 8%, option maturity is 10 years, and strike prices vary from 80 to 120.

<table>
<thead>
<tr>
<th>Risk aversion $\gamma$</th>
<th>Weight in options $\lambda$</th>
<th>Subjective value as a fraction of market value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BS</td>
</tr>
<tr>
<td>Panel A: At-the-money options (strike price = stock price)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>25.8</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>51.5</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>87.3</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>9.6</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>24.4</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>53.1</td>
</tr>
<tr>
<td>Panel B: In-the-money options (strike price/stock price = 0.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>32.0</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>59.5</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>94.3</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>12.3</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>29.9</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>60.8</td>
</tr>
<tr>
<td>Panel C: Out-of-the-money options (strike price/stock price = 1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>21.1</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>44.8</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>80.4</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>7.7</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>20.3</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>46.4</td>
</tr>
</tbody>
</table>
Table 3
Incentives to increase stock price

This table reports the incentives to increase stock price provided by the executive’s stock option grant. The incentives are measured by pay-to-performance sensitivity (PPS) defined as the ratio of the percentage change in the executive’s wealth over the percentage change in stock price. Comparison is made for four types of executive stock options: traditional stock options, Asian stock options, indexed stock options, and Asian indexed stock options. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected return is 8% for both the stock and the index, option maturity is 10 years, and strike prices vary from 25 to 125. The executive’s coefficient of relative risk aversion varies from 2 to 4 and his weight in options varies from 25% to 75%.
<table>
<thead>
<tr>
<th>Weight in options ((\lambda))</th>
<th>Strike price ((K))</th>
<th>Pay-to-performance sensitivity (PPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BS</td>
</tr>
<tr>
<td><strong>Panel A: Moderate level of risk aversion ((\gamma = 2))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>25</td>
<td>0.276</td>
</tr>
<tr>
<td>0.25</td>
<td>50</td>
<td>0.307</td>
</tr>
<tr>
<td>0.25</td>
<td>75</td>
<td>0.323</td>
</tr>
<tr>
<td>0.25</td>
<td>100</td>
<td>0.326</td>
</tr>
<tr>
<td>0.25</td>
<td>125</td>
<td>0.326</td>
</tr>
<tr>
<td>0.50</td>
<td>25</td>
<td>0.486</td>
</tr>
<tr>
<td>0.50</td>
<td>50</td>
<td>0.527</td>
</tr>
<tr>
<td>0.50</td>
<td>75</td>
<td>0.529</td>
</tr>
<tr>
<td>0.50</td>
<td>100</td>
<td>0.508</td>
</tr>
<tr>
<td>0.50</td>
<td>125</td>
<td>0.476</td>
</tr>
<tr>
<td>0.75</td>
<td>25</td>
<td>0.755</td>
</tr>
<tr>
<td>0.75</td>
<td>50</td>
<td>0.792</td>
</tr>
<tr>
<td>0.75</td>
<td>75</td>
<td>0.749</td>
</tr>
<tr>
<td>0.75</td>
<td>100</td>
<td>0.681</td>
</tr>
<tr>
<td>0.75</td>
<td>125</td>
<td>0.610</td>
</tr>
<tr>
<td><strong>Panel B: High level of risk aversion ((\gamma = 4))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>25</td>
<td>0.214</td>
</tr>
<tr>
<td>0.25</td>
<td>50</td>
<td>0.224</td>
</tr>
<tr>
<td>0.25</td>
<td>75</td>
<td>0.218</td>
</tr>
<tr>
<td>0.25</td>
<td>100</td>
<td>0.204</td>
</tr>
<tr>
<td>0.25</td>
<td>125</td>
<td>0.187</td>
</tr>
<tr>
<td>0.50</td>
<td>25</td>
<td>0.353</td>
</tr>
<tr>
<td>0.50</td>
<td>50</td>
<td>0.338</td>
</tr>
<tr>
<td>0.50</td>
<td>75</td>
<td>0.301</td>
</tr>
<tr>
<td>0.50</td>
<td>100</td>
<td>0.263</td>
</tr>
<tr>
<td>0.50</td>
<td>125</td>
<td>0.230</td>
</tr>
<tr>
<td>0.75</td>
<td>25</td>
<td>0.509</td>
</tr>
<tr>
<td>0.75</td>
<td>50</td>
<td>0.430</td>
</tr>
<tr>
<td>0.75</td>
<td>75</td>
<td>0.355</td>
</tr>
<tr>
<td>0.75</td>
<td>100</td>
<td>0.296</td>
</tr>
<tr>
<td>0.75</td>
<td>125</td>
<td>0.251</td>
</tr>
</tbody>
</table>
Fig. 1. The critical threshold of risk aversion level. These figures illustrate the executive’s critical threshold of risk aversion level ($\gamma^*$) at which he is indifferent between average stock price and stock price based equity incentives. At this critical level, both types of equity incentives have the same incentive effects and cost the firm exactly the same amount. In Panel A, the critical threshold is plotted against the stock’s excess return ($\mu - r$) while keeping the stock’s volatility ($\sigma$) fixed at 30%. In Panel B, the critical threshold is plotted against the stock’s volatility while keeping the stock’s excess return fixed at 4%.
Fig. 2. The executive’s subjective value of the stock option as a fraction of market value. This figure illustrates the executive’s subjective value of the stock option as a percentage of the option’s market value as option moneyness (strike price/stock price) varies from 0 to 2. This ratio measures the cost effectiveness of granting stock options. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional stock options (BS), Asian stock options, indexed stock options, and Asian indexed stock options.
Fig. 3. Incentives to increase stock price. This figure illustrates the incentives to increase stock price provided by the executive’s stock option grant. The incentives are measured by pay-to-performance sensitivity defined by the ratio of the percentage change in the executive’s wealth to the percentage change in stock price. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional stock options (BS), Asian stock options, indexed stock options, and Asian indexed stock options.
Fig. 4. Incentives to increase systematic risk. This figure illustrates the incentives to increase systematic risk (holding the idiosyncratic risk constant) provided by the executive’s stock option grant. The incentives are measured by a rescaled option vega defined as the percentage change in the executive’s wealth for a given percentage change in systematic risk ($\beta$). The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional (BS), Asian, indexed, and Asian indexed stock options.
Fig. 5. Incentives to increase idiosyncratic risk. This figure illustrates the incentives to increase idiosyncratic risk (holding the systematic risk constant) provided by the executive’s stock option grant. The incentives are measured by a rescaled option vega defined as the percentage change in the executive’s wealth for a given percentage change in idiosyncratic risk ($I = \frac{\sigma}{\beta \sigma^2}$). The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional (BS), Asian, indexed, and Asian indexed stock options.
Fig. 6. Incentives to increase total risk. This figure illustrates the incentives to increase total risk (holding the correlation between the stock and index constant) provided by the executive’s stock option grant. The incentives are measured by a rescaled option vega defined as the percentage change in the executive’s wealth for a given percentage change in total risk ($\sigma$). The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional (BS), Asian, indexed, and Asian indexed stock options.
Fig. 7. The executive’s subjective value of American options as a fraction of market value. This figure illustrates the executive’s subjective value of American stock options as a percentage of their market value as option moneyness (strike price/stock price) varies from 0 to 2. This ratio measures the cost effectiveness of granting stock options. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional stock options (BS), Asian stock options, indexed stock options, and Asian indexed stock options. A 400-period binomial tree model is used to value all American options.
Fig. 8. Incentives to increase stock price provided by American options. This figure illustrates the incentives to increase stock price provided by American stock options. The incentives are measured by pay-to-performance sensitivity defined by the ratio of the percentage change in the executive’s wealth to the percentage change in stock price. The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns (\( \rho \)) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. Comparison is made for four types of executive stock options: traditional stock options (BS), Asian stock options, American traditional stock options, and American Asian stock options. A 400-period binomial tree model is used to value all American options.
Fig. 9. Incentives to increase stock price: the impact of the executive’s outside investment opportunities. This figure illustrates the incentives to increase stock price provided by the executive’s stock option grant. The incentives are measured by pay-to-performance sensitivity defined by the ratio of the percentage change in the executive’s wealth to the percentage change in stock price. The incentives are calculated under the assumption that the executive’s outside wealth is invested in either the risk-free asset (with a subscript r) or the market portfolio (with a subscript m). The following parameters are used: the initial stock price is $100, the risk-free rate is 4%, the dividend yield is 2% for both the stock and index, volatility is 30% and 15% for the stock and index, respectively, the correlation coefficient between stock and index returns ($\rho$) is 0.5, the expected index return is 8%, option maturity is 10 years, and strike prices vary from 0 to 200. The executive’s coefficient of relative risk aversion is 3 and 50% of his wealth is tied to stock options. BS and Asian indicate traditional and Asian stock options, respectively.